

Astrophysics

Density of stars

1. Using $R_{\odot} = 6.96 \times 10^8$ m and $M_{\odot} = 1.989 \times 10^{30}$ kg, find $\bar{\rho}_{\odot}$ the average density of the Sun.
2. Compare this value to the density of water: 1g/cm^3 .
3. Calculate the density of Sirius, the brightest star in the sky with $2.3 M_{\odot}$ and $1.6 R_{\odot}$.
4. Calculate the density of Betelgeuse, use $10 M_{\odot}$ and $1000 R_{\odot}$ at its maximum. Compare this to the density of the air we breathe: At sea level and at 20°C dry air has a density of approximately 1.2 kg/m^3 . Betelgeuse is a tenuous ghostly object for which it is difficult to define what is meant by “surface”.
5. Calculate the density of Sirius B, white dwarf binary companion of Sirius. Use $1.053 M_{\odot}$ and $0.008 R_{\odot}$ (smaller than Earth). Electron degeneracy pressure is responsible for maintaining hydrostatic equilibrium in a white dwarf. Compare this to the densest material on Earth (not counting subatomic particles), iridium metal (22.65 g/cm^3).
6. Calculate the acceleration due to gravity at the surface of this star. Use $g = \frac{GM}{r^2}$.
7. Calculate the classical escape velocity at the surface of this star. Remember that the potential energy of a particle of mass m at a distance r from a massive object is of mass M is

$$U = -\frac{GMm}{r}$$
 and that to escape this particle must be able to reach infinity.
8. Neutron stars are held together by gravity and supported by neutron degeneracy pressure. Use $1.4 M_{\odot}$ and $R = 10$ km. Compare your result to the typical density of an atomic nucleus: $\rho_{nuc} \approx 2.3 \times 10^{17}\text{ kg/m}^3$.
9. Calculate the acceleration due to gravity at the surface of this star.
10. Calculate the classical escape velocity at the surface of this star.
11. For a star of mass M , calculate the classical radius for which the escape velocity becomes equal to the speed of light. This distance is called the *Schwarzschild radius*, beyond which a collapsing neutron star becomes a black hole, not even light can escape the gravitational attraction of the massively dense object.
12. Using the same method as above, calculate how close a photon can go close to a black hole of mass M , and still be able to escape. This distance is called the *event horizon* and nothing crossing this point can hope to escape the gravitational attraction of the black hole.