

Here is the solution to the twin paradox calculation using the Doppler effect:

We have two frames of reference:

frame A: the starship frame.

frame B: the Earth frame.

The traveller, blasts off from Earth towards Alpha-Centauri, 4.34 light-years away. This is the proper length (L_p) measured in B.

The travel distance measured in A is

$$L = \frac{L_p}{\gamma}$$

The time of total trip measured by B, the Earth frame is the distance traveled (there and back again) measured in B divided by the speed of the starship:

$$\Delta t_B = \frac{2L_p}{v}$$

(NOTE: if you divide the distance in lightyears by $\beta = v/c$, you get directly the time in years...)

The time of total trip measured by A, the starship frame is the distance traveled measured in A divided by the speed of the starship:

$$\Delta t_A = \frac{2L}{v}$$

The frequency of emission is set to 100 pulses a year for each observers in their own frame: $f = 100\text{yr}^{-1}$

The total Number of pulses emitted by observer in B is:

$$N_B = f \Delta t_B$$

The total Number of pulses emitted by observer in A is:

$$N_A = f \Delta t_A$$

The reduced frequency is measured as long as the starship has not turned around:

$$f' = f \left(\frac{1 - \beta}{1 + \beta} \right)$$

In here lies the loss of symmetry between the two observers, because A starts to receive the pulses coming from the Earth at an increased rate as soon as she turns around, however, the observer on Earth must wait for the first pulse sent by the starship at an increased rate to reach the Earth, 4.34 light-years away. Therefore:

Time of detecting A's turnaround in A is:

$$\Delta t_{1A} = \frac{L}{v}$$

Time of detecting A's turnaround in B is:

$$\Delta t_{1B} = \frac{L_p}{v} + \frac{L_p}{c}$$

The Number of signals received at reduced rate ;

in A:

$$N_{1A} = f' \Delta t_{1A}$$

in B:

$$N_{1B} = f' \Delta t_{1B}$$

The increased frequency is measured for the remainder of the trip:

$$f'' = f \left(\frac{1 + \beta}{1 - \beta} \right)$$

So the Number of signals received at an increased rate:

in A:

$$N_{2A} = f'' (\Delta t_A - \Delta t_{1A})$$

in B:

$$N_{2B} = f'' (\Delta t_B - \Delta t_{1B})$$

The total number of pulses received in A will be equal to the total number of pulses sent from B and vice versa, which proves that there is no paradox and that both observers agree that the person traveling will have seen less time elapsed than the stay-at-home.

Following are the numerical answers for a starship moving at 0.8c to Alpha-Centauri and back:

speed of starship = 0.800 c

gamma = 1.667

distance measured in B = 4.340 light years

trip time measured in B = 10.85 years, or 3960. days

distance measured in A = 2.604 light years

trip time measured in A = 6.510 years, or 2380. days

Total number of pulses sent in B = 1080. pulses

Total number of pulses sent in A = 651. pulses

Time of detecting A's turnaround in B = 9.7650 years

Time of detecting A's turnaround in A = 3.2550 years

reduced pulse rate = 33.3 yr⁻¹

increased pulse rate = 300. yr⁻¹

Number of signals received at reduced rate in B = 325. pulses

Number of signals received at reduced rate in A = 108. pulses

Number of signals received at increased rate in B = 326. pulses

Number of signals received at increased rate in A = 976. pulses

Total Number of pulses received in B = 651. pulses

Total Number of pulses received in A = 1080. pulses