

HEAT, WORK, AND THE FIRST LAW OF THERMODYNAMICS

18

EXERCISES

Section 18.1 The First Law of Thermodynamics

15. **INTERPRET** We identify the system as the water in the insulated container. The problem is about work done to raise the temperature of a system. The first law of thermodynamics is involved.

DEVELOP Since the container is perfectly insulated thermally, no heat enters or leaves the water in it. Thus, $Q = 0$ and the first law of thermodynamics in Equation 18.1 gives $\Delta U = Q - W = -W$. The change in the internal energy of the water is determined from its temperature rise, $\Delta U = mc \Delta T$ (see comments in Section 16.1 on internal energy).

EVALUATE The work done on the water is

$$W = -\Delta U = -mc \Delta T = -(1 \text{ kg})(4.184 \text{ kJ/kg} \cdot \text{K})(7 \text{ K}) = -29.3 \text{ kJ}$$

ASSESS The negative sign signifies that work was done on the water.

16. (a) The change in the internal energy of the water is $\Delta U = mc\Delta T$, and the work done by it (i.e., the negative of the work done on it) is given. Therefore, Equation 18.1 gives $Q = \Delta U + W = (0.5 \text{ kg})(4.184 \text{ kJ/kg} \cdot \text{K})(3 \text{ K}) + (-9.0 \text{ kJ}) = 6.28 \text{ kJ} - 9.0 \text{ kJ} = -2.72 \text{ kJ}$. (The negative sign signifies that the water lost heat to its surroundings.)
(b) If the water had been in perfect thermal isolation, no heat would have been transferred, $Q = 0$ and $W = -\Delta U = -6.28 \text{ kJ}$ instead of -9.0 kJ .

17. **INTERPRET** We identify the system as the gas that undergoes expansion. The problem is about the change of internal energy of a system and involves the first law of thermodynamics.

DEVELOP The heat added to the gas is $Q = Pt = (40 \text{ W})(25 \text{ s}) = 1000 \text{ J}$. In addition, the amount of work it does on its surrounding is $W = 750 \text{ J}$. The change in internal energy can be found by using the first law of thermodynamics given in Equation 18.1.

EVALUATE Using Equation 18.1 we find

$$\Delta U = Q - W = 1000 \text{ J} - 750 \text{ J} = 250 \text{ J}$$

ASSESS Since $\Delta U > 0$, we conclude that the internal energy has increased.

18. From Equation 18.2, $dQ/dt = dU/dt + dW/dt = 45 \text{ W} + 165 \text{ W} = 210 \text{ W}$.

19. **INTERPRET** This problem is about heat and mechanical energy, which are related by the first law of thermodynamics. The system is the automobile engine.

DEVELOP Since we are dealing with rates, we make use of Equation 18.2:

$$\frac{dU}{dt} = \frac{dQ}{dt} - \frac{dW}{dt}$$

If we assume that the engine system operates in a cycle, then $dU/dt = 0$. The engine's mechanical power output can then be calculated once the heat output is known.

EVALUATE The above conditions yield $(dQ/dt)_{\text{out}} = 68 \text{ kW}$ and $(dW/dt) = 0.17(dQ/dt)_{\text{in}}$. Equation 18.2 then gives

$$\frac{dW}{dt} = \frac{dQ}{dt} = \left(\frac{dQ}{dt}\right)_{\text{in}} - \left(\frac{dQ}{dt}\right)_{\text{out}} = \frac{1}{0.17} \frac{dW}{dt} - \left(\frac{dQ}{dt}\right)_{\text{out}}$$

or

$$\frac{dW}{dt} = \frac{(dQ/dt)_{\text{out}}}{(0.17)^{-1} - 1} = \frac{68 \text{ kW}}{(0.17)^{-1} - 1} = 13.9 \text{ kW}$$

ASSESS We find the mechanical power output dW/dt to be proportional to the heat output, $(dQ/dt)_{\text{out}}$. In addition, dW/dt also increases with the percentage of the total energy released in burning gasoline that ends up as mechanical work.

Section 18.2 Thermodynamic Processes

- 20.** The work done by the gas equals the area under the straight diagonal path AB in Fig. 18.19. The area of this trapezoid is $W = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(P_1 + 2P_1)(2V_1 - V_1) = \frac{3}{2}P_1V_1$. W can also be obtained from Equation 18.3. On the path AB , $P = P_1 + (V - V_1)(P_2 - P_1)/(V_2 - V_1)$. Then

$$W = \int_{V_1}^{V_2} PdV = P_1(V_2 - V_1) + \left(\frac{P_2 - P_1}{V_2 - V_1}\right) \frac{1}{2}(V_2 - V_1)^2 = \frac{3}{2}P_1V_1.$$

- 21. INTERPRET** The expansion of the ideal gas involves two stages: an isochoric (constant-volume) process and an isobaric (constant-pressure) process. We are asked to find the total work done by the gas.
DEVELOP For an isochoric process, $\Delta V = 0$ and $W = 0$. On the other hand, for an isobaric process, the work done is $W = p\Delta V$.

EVALUATE Path AC is isochoric, so $W_{AC} = 0$. Similarly, path CB is isobaric, so the work done during this stage is

$$W_{CB} = p_2(V_2 - V_1) = 2p_1(2V_1 - V_1) = 2p_1V_1$$

Thus, the total work done is $W_{ACB} = W_{AC} + W_{CB} = 0 + 2p_1V_1 = 2p_1V_1$.

ASSESS In the pV diagram, Fig. 18.19, the area under AC is zero, and that under CB , a rectangle, is $2p_1V_1$. The work done by the gas is the area under the pV curve.

- 22.** During an isothermal expansion, the work done by a given amount of ideal gas is $W = nRT \ln(V_2/V_1) = (0.3 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K})\ln(5) = 1.20 \text{ kJ}$ (see Equation 18.4).
23. INTERPRET The constant temperature of 300 K indicates that the process is isothermal.
DEVELOP We assume the gas to be ideal and apply the ideal-gas law given in Equation 17.1: $pV = NkT$. For an isothermal process, $T = \text{constant}$, we obtain $p_1V_1 = p_2V_2$. The total work done by the gas can be calculated using Equation 18.4:

$$W_{12} = nRT \ln\left(\frac{V_2}{V_1}\right)$$

EVALUATE (a) For the isothermal expansion process, the volume increases by a factor of

$$\frac{V_2}{V_1} = \frac{p_1}{p_2} = \frac{100 \text{ kPa}}{75 \text{ kPa}} = \frac{4}{3}$$

(b) Using Equation 18.4, the work done by the gas is

$$W_{12} = nRT \ln\left(\frac{V_2}{V_1}\right) = (0.3 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K})\ln\left(\frac{4}{3}\right) = 215 \text{ J}$$

ASSESS Since $V_2 > V_1$, we find the work to be positive, $W_{12} > 0$. This makes sense because the gas inside the balloon must do positive work to expand outward.

- 24.** In an isothermal compression of a fixed quantity of ideal gas, work is done on the gas so W is negative in Equation 18.4. For the values given, $W = nRT \ln(V_2/V_1) = (25 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K})\ln\left(\frac{1}{2}\right) = -4.32 \text{ kJ}$.

25. **INTERPRET** The thermodynamic process here is adiabatic, with no heat flowing between the system (the gas) and its environment.

DEVELOP In an adiabatic process, $Q = 0$, and the first law of thermodynamics becomes $\Delta U = -W$. The temperature and volume are related by Equation 18.11b:

$$TV^{\gamma-1} = \text{constant}$$

EVALUATE From the equation above, we have

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \rightarrow \frac{V_2}{V_1} = \left(\frac{T_1}{T_2} \right)^{1/(\gamma-1)}$$

Thus, for the temperature to double, the volume change is

$$\frac{V_2}{V_1} = \left(\frac{T_1}{T_2} \right)^{1/(\gamma-1)} = \left(\frac{1}{2} \right)^{1/(1.4-1)} = 0.177$$

ASSESS We see that increasing the temperature along the adiabat is accompanied by a volume decrease. In addition, since $pV^\gamma = \text{constant}$, the final pressure is also increased:

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right)^\gamma = p_1 \left(\frac{1}{0.177} \right)^{1.4} = 11.3 p_1$$

Section 18.3 Specific Heats of an Ideal Gas

26. Reasoning similar to that in Example 18.5 shows that

$$U = n_{\text{O}_2} N_A \frac{5}{2} kT + n_{\text{Ar}} N_A \frac{3}{2} kT = \left[\frac{5}{2}(2.5 \text{ mol}) + \frac{3}{2}(3 \text{ mol}) \right] RT = (10.75 \text{ mol})RT$$

Thus,

$$C_V = \frac{1}{n} \frac{dU}{dT} = \frac{(10.75 \text{ mol})R}{(2.5+3) \text{ mol}} = 1.95R \quad \text{and} \quad C_P = C_V + R = 2.95R$$

27. **INTERPRET** The problem is about the specific heat of a mixture of gases. We want to know what fraction of the molecules is monatomic.

DEVELOP The internal energy of a mixture of two ideal gases is

$$U = f_1 N \bar{E}_1 + f_2 N \bar{E}_2$$

where f_1 is the fraction of the total number of molecules, N , of type 1, and \bar{E}_1 is the average energy of a molecule of type 1, etc. Classically, $\bar{E} = g(\frac{1}{2}kT)$, where g is the number of degrees of freedom. The molar specific heat at constant volume is

$$C_V = \frac{1}{n} \frac{dU}{dT} = \frac{N_A}{N} \frac{d}{dT} \left(f_1 N g_1 \frac{1}{2} kT + f_2 N g_2 \frac{1}{2} kT \right) = \frac{1}{2} R (f_1 g_1 + f_2 g_2)$$

Suppose that the temperature range is such that $g_1 = 3$ for the monatomic gas, and $g_2 = 5$ for the diatomic gas, as discussed in Section 18.3. Then

$$C_V = \frac{1}{2} R (3f_1 + 5f_2) = R(2.5 - f_1)$$

where $f_2 = 1 - f_1$ since the sum of the fractions of the mixture is one. Now, C_V can also be specified by the ratio

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V} \rightarrow C_V = \frac{R}{\gamma - 1}$$

Equating the two expressions allows us to solve for f_1 .

EVALUATE Solving, we find $2.5 - f_1 = \frac{1}{\gamma-1} = \frac{1}{0.52}$, or $f_1 = 57.7\%$.

ASSESS From the equation above, we see that the specific-heat ratio can be written as

$$\gamma = 1 + \frac{1}{2.5 - f_1}$$

In the limit where all the gas molecules are monatomic, $f_1 = 1$, and $\gamma = 1.67$. On the other hand, if all the molecules are diatomic, then $f_1 = 0$ and the specific-heat ratio is $\gamma = 1.4$. The equation yields the expected results in both limits.

- 28.** Generalization of the result in the solution of the previous problem, gives the molar specific heat of a mixture of three gases: $C_V = f_1 C_{V1} + f_2 C_{V2} + f_3 C_{V3}$. For each gas and the mixture, use $C_V = R/(\gamma - 1)$ to obtain $(\gamma - 1)^{-1} = f_1(\gamma_1 - 1)^{-1} + f_2(\gamma_2 - 1)^{-1} + f_3(\gamma_3 - 1)^{-1}$. When the data specified in the problem is substituted, one gets $(\gamma - 1)^{-1} = 0.5(0.29)^{-1} + 0.3(0.4)^{-1} + 0.2(0.67)^{-1} = 2.77$, or $\gamma = 1.36$.
- 29. INTERPRET** The thermodynamic process is adiabatic, and we want to know the temperature change when work is done on the gas.
- DEVELOP** In an adiabatic process, $Q = 0$ so $\Delta U = -W$. From Equation 18.6, $\Delta U = nC_V \Delta T$, the change in temperature is

$$\Delta T = \frac{\Delta U}{nC_V} = \frac{-W}{nC_V}$$

Note that the change in internal energy is equal to the negative of the work done *by* the gas, or equivalently, the work done *on* the gas. If the work done per mole *on* the gas is $(-W)/n = 2.5$ kJ/mol, then $\Delta T = (2.5 \text{ kJ/mol})/C_V$.

EVALUATE (a) For an ideal monatomic gas, $C_V = \frac{3}{2}R = \frac{3}{2}(8.314 \text{ J/mol} \cdot \text{K})$, so $\Delta T = 200$ K.

(b) For an ideal diatomic gas (with five degrees of freedom), $C_V = \frac{5}{2}R$ so $\Delta T = 120$ K.

ASSESS Since the diatomic gas has a greater specific heat C_V , its temperature change is less than that of the monatomic gas.

PROBLEMS

- 30.** Consider the electric generator to be the system. It operates in a cycle, so its internal energy doesn't change $dU/dt = 0$ (otherwise the generator would store energy). The rate of mechanical energy input to the generator from the gravitational potential energy of falling water is $-(dm/dt)gy = -(10^6 \text{ kg/s})(9.8 \text{ m/s}^2)(50 \text{ m}) = -490$ MW (work done on the generator is negative), while the rate of work produced by the generator is 400 MW. Therefore, the first law of thermodynamics requires a heat flow of $dQ/dt = dW/dt = -490 \text{ MW} + 400 \text{ MW} = -90$ MW (negative for heat leaving the generator). This heat is absorbed by the water, causing a temperature rise satisfied by $(dm/dt)c \Delta T = 90$ MW. Thus $\Delta T = 90 \text{ MW}/(10^6 \text{ kg/s})(4.184 \text{ kJ/kg} \cdot \text{k}) = 2.15 \times 10^{-2} \text{ K} \approx 0.02 \text{ C}$.
- 31. INTERPRET** The constant temperature of 440 K indicates that the process is isothermal.
- DEVELOP** We apply the ideal-gas law given in Equation 17.1: $pV = NkT$. For an isothermal process, $T = \text{constant}$, we obtain $p_1V_1 = p_2V_2$. Since $\Delta U = 0$ for an isothermal process, the heat absorbed is equal to the total work done by the gas (Equation 18.4):

$$Q = W = nRT \ln \left(\frac{V_2}{V_1} \right)$$

EVALUATE (a) Using the equation above, the heat absorbed is $Q = W = 3.3$ kJ.

(b) Equation 18.4 gives

$$n = \frac{W}{RT \ln(V_2/V_1)} = \frac{3.3 \text{ kJ}}{(8.314 \text{ J/mol} \cdot \text{K})(440 \text{ K}) \ln 10} = 0.392 \text{ mol}$$

ASSESS The heat absorbed by the gas is equal to the work done by the gas on its surrounding as it expands, and there is no change in temperature.

- 32.** In an isothermal compression, work is done on the gas, so W in Equation 18.4 is negative. Thus $W = -61 \text{ J} = nRT \ln(V_2/V_1)$, or $T = -61 \text{ J}/(0.25 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K}) \ln(3.0/3.5) = 190 \text{ K}$.
- 33. INTERPRET** We take the air inside the spherical bubble to behave like an ideal gas at constant temperature. So the process is isothermal.

DEVELOP We apply the ideal-gas law given in Equation 17.1: $pV = NkT$. For an isothermal process, $T = \text{constant}$, we obtain $p_1V_1 = p_2V_2$. Since the volume of a spherical bubble of diameter d is $V = 4\pi(d/2)^3/3 = \pi d^3/6$, the relation between the diameter and the pressure can be written as

$$p_1 \left(\frac{\pi d_1^3}{6} \right) = p_2 \left(\frac{\pi d_2^3}{6} \right) \rightarrow \frac{d_2}{d_1} = \left(\frac{p_1}{p_2} \right)^{1/3}$$

EVALUATE (a) Using the equation above, we find the diameter at the maximum pressure to be

$$d_2 = \left(\frac{p_1}{p_2} \right)^{1/3} d_1 = \left(\frac{(80 + 760) \text{ mm of Hg}}{(125 + 760) \text{ mm of Hg}} \right)^{1/3} (1.52 \text{ mm}) = 1.49 \text{ mm}$$

(b) The work done *on* the air is the negative of Equation 18.4, or

$$\begin{aligned} W_{\text{on air}} &= -nRT \ln \left(\frac{V_2}{V_1} \right) = -p_1 V_1 \ln \left(\frac{p_1}{p_2} \right) = p_1 V_1 \ln \left(\frac{p_2}{p_1} \right) \\ &= (840 \text{ mm of Hg}) \left(\frac{101.3 \text{ kPa}}{760 \text{ mm of Hg}} \right) \frac{\pi}{6} (1.52 \text{ mm})^3 \ln \left(\frac{885 \text{ mm of Hg}}{885 \text{ mm of Hg}} \right) \\ &= 10.7 \text{ J} \end{aligned}$$

ASSESS Positive work is done by the blood in compressing the air bubble.

- 34.** For isothermal compressions starting from the same volume (and temperature), $W_{13}/W_{12} = \ln(V_3/V_1)/\ln(V_2/V_1)$ (see Equation 17.3). If $V_2 = V_1/2$, $V_3 = V_1/10$, and $W_{12} = -600 \text{ J}$, then $W_{13} = (-600 \text{ J})(\ln 10)/(\ln 2) = -1.99 \text{ kJ}$. [Note: $\ln x = -\ln(1/x)$.]

- 35. INTERPRET** The thermodynamic process here is adiabatic, with no heat flowing between the system (the gas) and its environment.

DEVELOP In an adiabatic process, $Q = 0$, and the first law of thermodynamics becomes $\Delta U = -W$. The pressure and volume are related by Equation 18.11a: $PV^\gamma = \text{constant}$. This implies

$$p_1 V_1^\gamma = p_2 V_2^\gamma \rightarrow \frac{p_2}{p_1} = \left(\frac{V_1}{V_2} \right)^\gamma$$

EVALUATE Taking the natural logarithm on both sides of the above to solve for γ , we obtain

$$\ln \left(\frac{p_2}{p_1} \right) = \gamma \ln \left(\frac{V_1}{V_2} \right) \rightarrow \gamma = \frac{\ln(p_2/p_1)}{\ln(V_1/V_2)} = \frac{\ln 2.55}{(\ln 2)} = 1.35$$

ASSESS The value of γ indicates that gas consists of polyatomic molecules.

- 36.** The work done *by* an ideal gas undergoing an adiabatic process is $W_{12} = (P_1V_1 - P_2V_2)/(\gamma - 1)$ (see Equation 18.12). Since the compression is specified by given values of P_1 , V_1 , and V_2 , we first find the final pressure from the adiabatic gas law. **(b)** $P_2 = P_1(V_1/V_2)^\gamma = (100 \text{ kPa})(5 \text{ L}/2.5 \text{ L})^{1.4} = 264 \text{ kPa}$. **(a)** Then the work done *on* the gas (which is $-W_{12}$) is $-W_{12} = (P_2V_2 - P_1V_1)/(\gamma - 1) = [(264 \text{ kPa})(2.5 \text{ L}) - (100 \text{ kPa})(5 \text{ L})]/0.4 = 399 \text{ J}$.
- 37. INTERPRET** The problem involves a cyclic process. The three processes that make up the cycle are: isothermal (AB), isochoric (BC), and isobaric (CA).

DEVELOP Along the isotherm AB where $T = \text{constant}$, we have $p_A V_A = p_B V_B$. For an isothermal process, the work done by the gas is (Equation 18.4):

$$W = Q = nRT \ln \left(\frac{V_2}{V_1} \right)$$

EVALUATE (a) If AB is an isotherm, with $V_A = 5 \text{ L}$ and $V_B = 1 \text{ L}$, then the ideal gas law gives

$$P_B = \left(\frac{V_A}{V_B} \right) P_A = \left(\frac{5}{1} \right) (60 \text{ kPa}) = 300 \text{ kPa}$$

(b) The work done *by* the gas in the isothermal process AB is

$$W_{AB} = nRT_A \ln\left(\frac{V_B}{V_A}\right) = p_A V_A \ln\left(\frac{V_B}{V_A}\right) = (300 \text{ J}) \ln\left(\frac{1}{5}\right) = -483 \text{ J}$$

The process BC is isochoric and $W_{BC} = 0$. Similarly, the process CA is isobaric so

$$W_{CA} = p_A (V_A - V_C) = (60 \text{ kPa})(5 \text{ L} - 1 \text{ L}) = 240 \text{ J}$$

The total work done by the gas is

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA} = -483 \text{ J} + 0 + 240 \text{ J} = -243 \text{ J}$$

The work done *on* the gas is the negative of this, that is, $W_{\text{on gas}} = 243 \text{ J}$.

ASSESS Since the process is cyclic, the system returns to its original state, there's no net change in internal energy, and $\Delta U = 0$. This implies that $Q = W_{ABCA} = -243 \text{ J}$. That is, 243 J of heat must come *out* of the system.

38. (a) If AB represents an adiabatic process for an ideal gas, then the adiabatic law and the given values yield $P_B = P_A (V_A/V_B)^\gamma = (60 \text{ kPa})(5)^{1.4} = 571 \text{ kPa}$. (b) The work done *by* the gas over the adiabat AB is $W_{AB} = (P_A V_A - P_B V_B)/(\gamma - 1) = [(60 \text{ kPa})(5 \text{ L}) - (571 \text{ kPa})(1 \text{ L})]/0.4 = -678 \text{ J}$ (see Equation 18.12). The process BC is isovolumic so $W_{BC} = 0$, and the process CA is isobaric so $W_{CA} = P_A (V_A - V_C) = (60 \text{ kPa})(5 \text{ L} - 1 \text{ L}) = 240 \text{ J}$. The total work done by the gas is $W_{ABCA} = W_{AB} + W_{BC} + W_{CA} = -678 \text{ J} + 0 + 240 \text{ J} = -438 \text{ J}$. The work done *on* the gas is the negative of this.

39. **INTERPRET** We identify the thermodynamic process here as adiabatic compression.

DEVELOP In an adiabatic process, $Q = 0$, and the first law of thermodynamics becomes $\Delta U = -W$. The temperature and volume are related by Equation 18.11b:

$$TV^{\gamma-1} = \text{constant}$$

From the equation above, we obtain

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

where V_1/V_2 is the compression ratio (for T and V at maximum compression).

EVALUATE Substituting the values given, we have

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (303 \text{ K})(8.5)^{0.4} = 713 \text{ K} = 440 \text{ C}$$

(Note: T appearing in the gas laws is the absolute temperature.)

ASSESS The higher the compression ratio V_1/V_2 , the greater the temperature at the maximum compression, and hence a higher thermal efficiency.

40. $V/V_0 = (P_0/P)^{1/\gamma} = (0.5)^{1/1.4} = 0.610$ (Equation 18.11a).

41. **INTERPRET** We identify the thermodynamic process here as adiabatic compression.

DEVELOP In an adiabatic process, $Q = 0$, and the first law of thermodynamics becomes $\Delta U = -W$. The temperature and volume are related by Equation 18.11b:

$$TV^{\gamma-1} = \text{constant}$$

From the equation above, we obtain

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

where V_1/V_2 is the compression ratio (for T and V at maximum compression). In addition, since $pV^\gamma = \text{constant}$, the final pressure is $p_2 = p_1 (V_1/V_2)^\gamma$.

EVALUATE (a) Substituting the values given in the problem statement, we find the air temperature at the maximum compression to be

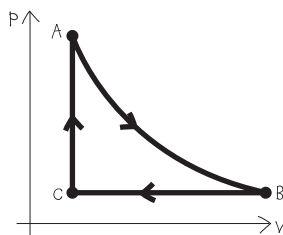
$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (320 \text{ K})(10.2)^{0.4} = 810 \text{ K}$$

(b) The corresponding pressure is

$$p_2 = p_1(V_1/V_2)^\gamma = (101.3 \text{ kPa})(10.2)^{1.4} = 2.62 \text{ MPa} = 25.8 \text{ atm}$$

ASSESS The higher the compression ratio V_1/V_2 , the greater the temperature and pressure at the maximum compression, and hence, a higher thermal (fuel) efficiency.

42. (a) In the PV diagram shown, AB is an isotherm (given by $PV = \text{constant} = P_A V_A = P_B V_B$, with $V_B > V_A$ for an expansion), BC is a straight horizontal line ($P = \text{constant} = P_B = P_C$), and CA is a vertical line ($V = \text{constant} = V_C = V_A$). (b) $W_{\text{tot}} = W_{AB} + W_{BC} + W_{CA}$. The first two terms are given, since for the isotherm, $Q_{AB} = W_{AB} = 35 \text{ J}$, and $W_{BC} = -22 \text{ J}$ is the work done by the gas, while $W_{CA} = 0$ for an isovolumic process. Thus $W_{\text{tot}} = 35 \text{ J} - 22 \text{ J} + 0 = 13 \text{ J}$, positive for work done by the gas. (c) For the whole cycle, $\Delta U_{\text{tot}} = 0$, so $Q_{\text{tot}} = W_{\text{tot}}$. Since $Q_{\text{tot}} = Q_{AB} + Q_{BCA}$, we find that $Q_{BCA} = Q_{\text{tot}} - Q_{AB} = W_{\text{tot}} - Q_{AB} = 13 \text{ J} - 35 \text{ J} = -22 \text{ J}$. Negative Q_{BCA} means heat leaves the gas during the process BCA .



43. **INTERPRET** This problem is to explore how different ways of adding heat (isothermal, isochoric, or isobaric) affects the final temperature of the system.

DEVELOP In an isothermal process, the temperature T is kept constant. With $\Delta U = 0$, the first law of thermodynamics gives $W = Q$. In an isochoric process, $\Delta V = 0$, and $W = 0$. First law of thermodynamics gives $Q = \Delta U = nC_V \Delta T$. Finally, in an isobaric process, $\Delta p = 0$ and

$$Q = nC_p \Delta T = n(C_V + R)\Delta T$$

These are the equations we shall use to solve for ΔT and W in each case.

EVALUATE (a) When heat is added isothermally, T is constant, so the final temperature is $T_2 = 300 \text{ K}$. Since $\Delta U = 0$, $W = Q = 1.5 \text{ kJ}$.

(b) From the above, we see that in an isochoric process, $W = 0$ and

$$\Delta T = \frac{Q}{nC_V} = \frac{1.5 \text{ kJ}}{(2 \text{ mol})(\frac{5}{2} R)} = 36.1 \text{ K}$$

Therefore, $T_2 = 300 \text{ K} + \Delta T = 336 \text{ K}$.

(c) In an isobaric process,

$$\Delta T = \frac{Q}{nC_p} = \frac{Q}{n(C_V + R)} = \frac{1.5 \text{ kJ}}{(2 \text{ mol})(\frac{5}{2} R + R)} = 25.8 \text{ K}$$

and $T_2 = 326 \text{ K}$. The work done is

$$W = p\Delta V = nR\Delta T = \frac{R}{C_p} Q = \frac{R}{(7R/2)} Q = 429 \text{ J}$$

ASSESS Comparing all three cases, we find

$$\Delta T: \text{isothermal} < \text{isobaric} < \text{isochoric}$$

$$W: \text{isochoric} < \text{isobaric} < \text{isothermal}$$

The results agree with that illustrated in Table 18.1.

44. The equations of an adiabat and an isotherm, passing through the point (V_0, P_0) in the PV diagram are $P(V) = P_0 V_0^\gamma / V^\gamma$ and $P(V) = P_0 V_0 / V$, respectively. The slopes are $dP/dV = P_0 V_0^\gamma (-\gamma V^{-\gamma-1}) = -\gamma P/V$ for the adiabat, and $P_0 V_0 (-V^{-2}) = -P/V$ for the isotherm, as stated.
45. **INTERPRET** The problem involves a cyclic process, and we identify three separate stages of the cycle: adiabatic, isochoric, and isothermal.
- DEVELOP** In an adiabatic process (AB), $Q = 0$, and the first law of thermodynamics becomes $\Delta U = -W$. The pressure and volume are related by Equation 18.11a: $pV^\gamma = \text{constant}$. This implies

$$P_A V_A^\gamma = P_B V_B^\gamma \rightarrow P_B = \left(\frac{V_A}{V_B}\right)^\gamma P_A$$

Point C lies on an isotherm with A , so the ideal-gas law (Equation 17.1) yields

$$P_C = \frac{P_A V_A}{V_C}$$

EVALUATE (a) From the equation above, the pressure at point B is

$$P_B = \left(\frac{V_A}{V_B}\right)^\gamma P_A = (250 \text{ kPa}) \left(\frac{1}{3}\right)^{1.67} = 39.9 \text{ kPa}$$

(b) The pressure at point C is $p_C = P_A \frac{V_A}{V_C} = (250 \text{ kPa}) \left(\frac{1}{3}\right) = 83.3 \text{ kPa}$.

(c) The net work done by the gas is $W_{ABCA} = W_{AB} + W_{BC} + W_{CA}$. W_{AB} is for an adiabatic process (Equation 18.12) and equals

$$W_{AB} = \frac{P_A V_A - P_B V_B}{\gamma - 1} = \frac{(250 \text{ kPa})(1 \text{ m}^3) - (39.9 \text{ kPa})(3 \text{ m}^3)}{0.67} = 194 \text{ kJ}$$

W_{BC} is for an isochoric process and equals zero. Finally, W_{CA} is for an isothermal process (Equation 18.4) and equals

$$W_{CA} = nRT_A \ln\left(\frac{V_A}{V_C}\right) = (250 \text{ kJ}) \ln\left(\frac{1}{3}\right) = -275 \text{ kJ}$$

Thus,

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA} = 194 \text{ kJ} + 0 + (-275 \text{ kJ}) = -80.2 \text{ kJ}$$

The work done on the gas is the negative of this.

ASSESS Since the process is cyclic, the system returns to its original state, there's no net change in internal energy, and $\Delta U = 0$. This implies that $Q = W_{ABCA} = -80.2 \text{ kJ}$. That is, 80.2 kJ of heat must come out of the system.

46. In this problem, the PV diagram is different from Fig. 18.14; just point A is the same. (a) As in Example 18.4, $W_{AB} = P_A V_A [1 - (V_A/V_B)^{\gamma-1}] / (\gamma - 1) = (400 \text{ J})(1 - 2^{0.4}) / 0.4 = -320 \text{ J}$. Now, BC is isobaric, so $W_{BC} = P_B (V_C - V_B)$. But $P_B = P_A (V_A/V_B)^\gamma$ so $P_B V_B = P_A V_A (V_A/V_B)^{\gamma-1}$ and C is at the same temperature as A so $P_C V_C = P_B V_C = P_A V_A$. Therefore, $W_{BC} = P_A V_A [1 - (V_A/V_B)^{\gamma-1}] = (400 \text{ J})(1 - 2^{0.4}) = -128 \text{ J}$. CA is still isothermal so $W_{CA} = P_A V_A \ln(V_A/V_C)$. But $V_C = P_A V_A / P_B = (V_B/V_A)^{\gamma-1} V_B = \left(\frac{1}{2}\right)^{0.4} (2 \text{ L}) = 1.52 \text{ L}$, which is the minimum volume called for in part (b) (see above), so $W_{CA} = (400 \text{ J}) \ln(4/1.52) = 388 \text{ J}$. Finally $W_{\text{net}} = -59.1 \text{ J}$, and the net work done on the gas is the negative of this.
47. **INTERPRET** We find the work done in a given heat cycle. In each part of the cycle, the work done is the area under the p - V curve.
- DEVELOP** The gas is taken through four parts of a cycle.
- (a) It is heated at constant volume until the pressure is doubled.
- (b) It is compressed adiabatically until its volume is $\frac{1}{4}$ the initial value.
- (c) It is cooled at constant volume to a temperature of 300 K.
- (d) It is expanded isothermally until it returns to the original state.
- Parts (a) and (c) do no work, as the volume is constant.

The work in part (b) is the area under an adiabatic curve, $W_b = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$. In order to use this, we will need the final pressure, which we can obtain by using $pV^\gamma = \text{constant}$.

The work in part (d) is the area under an isothermal curve, $W_d = nRT \ln\left(\frac{V_2}{V_1}\right)$. We will need to know the number of moles n , which we can obtain from $pV = nRT$.

The total work is the sum $W = W_b + W_d$.

We are told that the gas is an ideal gas with $\gamma = 1.4$, it has a volume $V_1 = 4.0 \text{ L} = 4 \times 10^{-3} \text{ m}^3$ at $T = 300 \text{ K}$, and $p_1 = 100 \text{ kPa}$.

EVALUATE The pressure during part (b) changes from $2p_1$ to $(2p_1)V_1^\gamma = p_2 V_2^\gamma \rightarrow p_2 = 2p_1 \left(\frac{V_1}{V_2}\right)^\gamma = 2p_1 (4)^\gamma$. The work for part (b) is then $W_b = \frac{2p_1 V_1 - 2p_1 4^\gamma \left(\frac{1}{4}V_1\right)}{\gamma - 1} = \frac{2p_1 V_1}{\gamma - 1} (1 - 4^{\gamma-1})$.

The number of moles of gas is $n = \frac{pV}{RT}$, and the work for part (d) is $W_d = nRT \ln\left(\frac{V_f}{V_i}\right) = \left(\frac{p_1 V_1}{RT}\right) RT \ln\left(\frac{V_1}{\frac{1}{4}V_1}\right) = p_1 V_1 \ln(4)$.

The total work is then $W_b + W_d = \frac{2p_1 V_1}{\gamma - 1} (1 - 4^{\gamma-1}) W_d + p_1 V_1 \ln(4) = p_1 V_1 \left[\frac{2(1 - 4^{\gamma-1})}{\gamma - 1} + \ln 4 \right] = -928 \text{ J}$.

ASSESS The work done is negative. If we draw a p-V diagram of this motion, we can see that the area under the curve is negative, since the gas goes around the cycle “counterclockwise.”

48. The individual segments of the process are the same as in Example 18.4, except that the order of states is reversed, i.e., ACBA in Fig. 18.14. The parameters of states B and C can be related to those of state A, by the adiabatic and ideal gas laws, as in the solutions to Problems 45 and 46. W_{AC} (isothermal) $= P_A V_A \ln(V_C/V_A) = (50 \text{ kPa})(25 \text{ L}) \ln\left(\frac{1}{3}\right) = -1.37 \text{ kJ}$. W_{CB} (isovolumic) $= 0$. W_{BA} (adiabatic) $= (P_B V_B - P_A V_A)/(\gamma - 1) = P_A V_A [(V_A/V_B)^{\gamma-1} - 1]/(\gamma - 1) = (1.25 \text{ kJ})(3^{0.67} - 1)/0.67 = 2.03 \text{ kJ}$. Thus $W_{\text{net}} = W_{AC} + W_{CB} + W_{BA} = 656 \text{ J}$, a positive value indicating work done by the gas on its surroundings.

49. **INTERPRET** The problem involves a cyclic process, and we identify three separate stages of the cycle: adiabatic, isochoric, and isothermal.

DEVELOP In an adiabatic process (AB), $Q = 0$, and the first law of thermodynamics becomes $\Delta U = -W$. The pressure and volume are related by Equation 18.11a: $pV^\gamma = \text{constant}$, and the work done by the gas is

$$W_{AB} = \frac{P_A V_A - P_B V_B}{\gamma - 1}$$

Since $\Delta V_{BC} = 0$ for the isochoric process, $W_{BC} = 0$. Similarly, for an isothermal process, the work done is (Equation 18.4)

$$W_{CA} = nRT_A \ln\left(\frac{V_A}{V_C}\right)$$

The minimum volume attained is V_B , which is the volume at the end of the adiabatic compression. Note that $V_B = V_C$ since $\Delta V_{BC} = 0$.

EVALUATE The solutions below are presented in the reverse order.

(c) The same individual processes applied in this problem are in the same order as in Example 18.4, so the pV diagram looks just like Figure 18.14, except that $V_A = 25 \text{ L}$ and $P_B = 3P_A = 3(50 \text{ kPa})$.

(b) The minimum volume attained can be found from the adiabatic law (Equation 18.11a):

$$V_C = V_B = V_A \left(\frac{P_A}{P_B}\right)^{1/\gamma} = (25 \text{ L}) \left(\frac{1}{3}\right)^{1/1.67} = 12.9 \text{ L}$$

(a) The work done *on* the gas is the negative of the work done *by* the gas:

$$\begin{aligned} W_{\text{on gas}} &= -W_{ABCA} = -(W_{AB} + W_{BC} + W_{CA}) = -\frac{p_A V_A - p_B V_B}{\gamma - 1} - 0 - P_A V_A \ln\left(\frac{V_A}{V_C}\right) \\ &= -\frac{(50 \text{ kPa})(25 \text{ L}) - (150 \text{ kPa})(12.9 \text{ L})}{1.67 - 1} - (50 \text{ kPa})(25 \text{ L}) \ln\left(\frac{25 \text{ L}}{12.9 \text{ L}}\right) \\ &= 211 \text{ J} \end{aligned}$$

since *AB* is adiabatic, *BC* is isochoric, and *CA* is isothermal.

ASSESS Since the process is cyclic, the system returns to its original state, there's no net change in internal energy, and $\Delta U = 0$. This implies that $Q = W_{ABCA} = -211 \text{ J}$. That is, 211 J of heat must come *out* of the system.

50. If no heat is lost (or gained) by the gas, the compression is adiabatic and Equation 18.11b gives $TV^{\gamma-1} = T_0 V_0^{\gamma-1}$. Therefore, the temperature rise is $T - T_0 = \Delta T = T_0[(V_0/V)^{\gamma-1} - 1]$. Since $V_0/V = (30 \text{ cm}^3/17 \text{ cm}^3)$, $\Delta T = [(30/17)^{0.4} - 1](293 \text{ K}) = 74.7 \text{ C}$.

51. **INTERPRET** The volume of the flask does not change, so we identify the thermodynamic process as isochoric. **DEVELOP** In an isochoric process, $\Delta V = 0$, and $W = 0$. The first law of thermodynamics (Equation 18.1) gives $Q = \Delta U = nC_V \Delta T$.

EVALUATE Substituting the values given, we find the heat that must be added to raise the temperature to be

$$Q = nC_V \Delta T = \left(\frac{p_0 V_0}{RT_0}\right)(2.5R)\Delta T = \frac{(100 \text{ kPa})(5 \text{ L})}{R(273 \text{ K})}(2.5R)(20 \text{ K}) = 91.6 \text{ J}$$

where we have used the ideal gas law to find n .

ASSESS Adding the heat raises the air temperature inside the flask. Since the volume does not change, by ideal-gas law, the pressure must also go up. The final pressure can be calculated as

$$p = \frac{nRT}{V_0} = \frac{nRT_0}{V_0} \frac{T}{T_0} = p_0 \frac{T}{T_0} = (100 \text{ kPa}) \frac{293 \text{ K}}{273 \text{ K}} = 107 \text{ kPa}$$

52. The work done *on* the gas is $-W = \Delta U - Q = nC_V \Delta T - Q = (21 \text{ mol})\left(\frac{3}{2} \times 8.314 \text{ J/mol} \cdot \text{K}\right)(160 \text{ K}) - (-15 \text{ kJ}) = 56.9 \text{ kJ}$.

53. **INTERPRET** The thermodynamic process here involves two stages: isothermal compression followed by an adiabatic compression.

DEVELOP During the first stage, since the gas is compressed isothermally, $\Delta T = 0$, there is no change in the temperature of the system, but the volume is reduced from V_0 to $V_1 = V_0/3$. During the next stage of adiabatic compression, $Q = 0$, and the first law of thermodynamics becomes $\Delta U = -W$. The temperature and volume are related by Equation 18.11b:

$$TV^{\gamma-1} = \text{constant}$$

which gives

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

The final temperature is T_2 .

EVALUATE Substituting the values given, we find T_2 to be

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (273 \text{ K}) \left(\frac{V_0/3}{V_0/5}\right)^{(7/5)-1} = (273 \text{ K}) \left(\frac{5}{3}\right)^{0.4} = 335 \text{ K}$$

ASSESS Since $TV^{\gamma-1} = \text{constant}$ for an adiabatic process, the compression results in an increase of the final temperature.

54. $pV^\gamma = p^{1-\gamma}(pV)^\gamma = p^{1-\gamma}(nRT)^\gamma$, so $p^{1-\gamma}T^\gamma = \text{constant}$. This can also be expressed as $p^{(1-\gamma)/\gamma}T = \text{constant}$, or $pT^{\gamma/(1-\gamma)} = \text{constant}$, where the constants are all different, of course. This gives $T = T_0(P_0/P)^{(1/\gamma)-1} = (273 \text{ K})(100/240)^{(1/1.3)-1} = 334 \text{ K}$.

55. INTERPRET The problem involves a cyclic process, and we identify three separate stages of the cycle: isochoric (AB), isobaric (BC), and isothermal (CA).

DEVELOP The work done *by* the gas in each segment of the cycle is summarized in Table 18.1. For the isochoric process (path AB), $\Delta V = 0$, and $W = 0$. For the isobaric process (path BC), the work done is

$$W_{BC} = p_B(V_C - V_B)$$

Finally, for the isothermal process (CA), the work done is (Equation 18.4)

$$W_{CA} = nRT_A \ln\left(\frac{V_A}{V_C}\right)$$

EVALUATE (a) Using the equations above, we obtain

$$W_{AB} = 0 \quad (\text{isochoric})$$

$$W_{BC} = p_B(V_C - V_B) = (250 \text{ kPa})(1 \text{ L} - 5 \text{ L}) = -1000 \text{ J} \quad (\text{isobaric})$$

$$W_{CA} = nRT_A \ln\left(\frac{V_A}{V_C}\right) = p_A V_A \ln\left(\frac{V_A}{V_C}\right) = (50 \text{ kPa})(5 \text{ L}) \ln(5) = 402 \text{ J} \quad (\text{isothermal})$$

Adding up all the contributions, the net work done *by* the gas is

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA} = 0 - 1000 \text{ J} + 402 \text{ J} = -598 \text{ J}$$

The net work done *on* the gas is simply the negative of this, $W_{\text{on gas}} = -W_{ABCA} = 598 \text{ J}$.

(b) With V held constant, the heat transferred is

$$Q_{AB} = nC_V \Delta T = \frac{nR(T_B - T_A)}{\gamma - 1} = \frac{p_B V_B - p_A V_A}{\gamma - 1} = \frac{(250 \text{ kPa} - 50 \text{ kPa})(5 \text{ L})}{1.4 - 1} = 2.50 \text{ kJ}$$

Since $Q_{AB} > 0$, heat is transferred into the gas.

ASSESS At constant volume, the gas must be heated in order to raise its pressure. In the above, we have used $C_V = R/(\gamma - 1)$. The equation can be derived as follows:

$$\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} = 1 + \frac{R}{C_v} \rightarrow C_v = \frac{R}{\gamma - 1}$$

56. (a) Now, $-W_{ACDA} = P_A V_A \ln(V_A/V_C) + 0 + P_A(V_D - V_A) = (50 \text{ kPa})(5 \text{ L}) \ln(5) - (50 \text{ kPa})(4 \text{ L}) = 202 \text{ J}$. **(b)** $Q_{CD} = nC_V \Delta T = nR(T_D - T_C)/(\gamma - 1) = (P_D - P_C)V_C/(\gamma - 1) = (-200 \text{ kPa})(1 \text{ L})/0.4 = -500 \text{ J}$, negative for heat transferred out of the gas (at constant volume, the gas must be cooled to lower its pressure).

57. INTERPRET We identify the thermodynamic process as adiabatic expansion of a gas mixture.

DEVELOP In an adiabatic process (AB), $Q = 0$, and the pressure and volume are related by Equation 18.11a: $pV^\gamma = \text{constant}$. This implies

$$p_1 V_1^\gamma = p_2 V_2^\gamma \rightarrow \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^\gamma$$

Taking the natural logarithm on both sides of the above to solve for γ , we obtain

$$\ln\left(\frac{p_2}{p_1}\right) = \gamma \ln\left(\frac{V_1}{V_2}\right) \rightarrow \gamma = \frac{\ln(p_2/p_1)}{\ln(V_1/V_2)} = \frac{\ln(1/3)}{\ln(1/2)} = \frac{-\ln(3)}{-\ln(2)} = 1.58$$

To find the fraction of the molecules that are argon, we use the result from Exercise 27:

$$f_1 = 2.5 - \frac{1}{\gamma - 1}$$

EVALUATE Substituting $\gamma = 1.58$ into the equation above gives

$$f_{\text{Ar}} = 2.5 - \frac{1}{\gamma - 1} = 2.5 - \frac{1}{1.58 - 1} = 0.790 = 79.0\%$$

ASSESS In the limit where all the gas molecules are monatomic, $f_1 = 1$, and $\gamma_{\text{monatomic}} = 1.67$. On the other hand, if all the molecules are diatomic, then $f_1 = 0$ and the specific-heat ratio is $\gamma_{\text{diatomic}} = 1.4$. Our ratio of $\gamma = 1.58$ is closer to 1.67. This implies that the gas mixture is predominantly monatomic.

58. The specific heat of a mixture of two gases is $C_V = f_1 C_{V1} + f_2 C_{V2}$, where the f 's are the number fractions of the gases. If gas 1 is monatomic ($C_V = \frac{3}{2}R$), gas 2 is triatomic (with $C_V = 3R$, as described in the text following Example 18.5), and we wish the mixture to have $C_V = \frac{5}{2}R$ appropriate to a diatomic gas, then $\frac{5}{2}R = \frac{3}{2}Rf_1 + 3Rf_2$, or $5 = 3f_1 + 6f_2$. Since $f_1 + f_2 = 1$, one finds $f_1 = \frac{1}{3}$ and $f_2 = \frac{2}{3}$. With 10 mol of gas 1, one needs 20 mol of gas 2.

59. **INTERPRET** This problem is about melting, and it involves heat of fusion. The source of energy is the mechanical energy of the rock.

DEVELOP The mechanical energy of the rock (originally gravitational potential energy) melted the ice (changed its internal energy) and no heat energy was transferred ($Q = 0$).

EVALUATE From the first law of thermodynamics, we have

$$-W = \Delta U \rightarrow m_{\text{rock}}gh = m_{\text{ice}}L_f$$

Thus, we find the height from which the rock is dropped to be

$$h = \frac{m_{\text{ice}}L_f}{m_{\text{rock}}g} = \frac{(0.0063 \text{ kg})(334 \text{ J/g})}{(8.5 \text{ kg})(9.8 \text{ m/s}^2)} = 25.3 \text{ m}$$

ASSESS In this problem, the rock did positive work on the ice-water system. Therefore, the work done *by* the system is negative, $W < 0$.

60. For an isothermal expansion of an ideal gas, Equation 18.4 and the ideal gas law give: $W = Q = nRT \ln(V_2/V_1) = nRT \ln(P_1/P_2)$, (since $PV = \text{constant}$), or $P_1 = P_2 e^{Q/nRT}$. The heat Q , supplied by the ice-water bath, is equal to the heat released when 10 g of water freezes at 0°C , i.e., $Q = (0.01 \text{ kg})(334 \text{ kJ/kg}) = 3.34 \text{ kJ}$. Thus, $P_1 = (1 \text{ atm})\exp\{(3.34 \text{ kJ})/(0.3 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(273 \text{ K})\} = (1 \text{ atm})e^{4.91} = 135 \text{ atm}$.

61. **INTERPRET** We identify the thermodynamic process as isobaric, with pressure being kept at 1.0 atm.

DEVELOP Since the process is isobaric ($p = 1 \text{ atm}$), the work done is $W = p\Delta V$. The initial volume of 1 g of water at 50°C is approximately $V_1 = 1 \text{ mL}$, while the final volume of 1 g of steam at 200°C (from the ideal gas law) is

$$V_2 = \frac{nRT_2}{p} = \frac{(\frac{1}{18} \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(473 \text{ K})}{1.013 \times 10^5 \text{ Pa}} = 2.16 \text{ L}$$

On the other hand, the heat added (at constant pressure) is

$$Q = mc_{\text{water}}(100 \text{ C} - 50 \text{ C}) + mL_v + nC_p(200 \text{ C} - 100 \text{ C})$$

which includes raising the temperature of the water, changing its state to steam, and raising the temperature of the steam.

EVALUATE (a) Substituting the values given, we find the work done by the system to be

$$W = p\Delta V = p(V_2 - V_1) = (1 \text{ atm})(2.16 - 0.001)\text{L}(101.3 \text{ J/L} \cdot \text{atm}) = 218 \text{ J}$$

(b) Using $C_p = C_V + R = 5.3R$, and values from Tables 16.1 and 17.1, we find

$$\begin{aligned} Q &= (1 \text{ g})(4.184 \text{ J/g} \cdot \text{K})(50 \text{ K}) + 2257 \text{ J} + \left(\frac{1}{18} \text{ mol}\right)(5.3)(8.314 \text{ J/mol} \cdot \text{K})(100 \text{ K}) \\ &= 0.209 \text{ kJ} + 2.257 \text{ kJ} + 0.245 \text{ kJ} = 2.711 \text{ kJ} \end{aligned}$$

ASSESS Most of the heat added goes to changing the phase of the system from water to steam. By first law of thermodynamics, the internal energy of the system has increased by $\Delta U = Q - W$.

62. The work done by an ideal gas undergoing an adiabatic process from state 1 to state 2 can be found by integration of the adiabatic law, $P = P_1 V_1^\gamma / V^\gamma$.

$$W_{12} = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} (P_1 V_1^\gamma) \frac{dV}{V^\gamma} = P_1 V_1^\gamma \left(\frac{V_2^{-\gamma+1} - V_1^{-\gamma+1}}{-\gamma+1} \right) = (P_1 V_1^\gamma V_1^{-\gamma+1} - P_2 V_2^\gamma V_2^{-\gamma+1})/(\gamma-1)$$

which is Equation 18.12. (Note: $P_1 V_1^\gamma = P_2 V_2^\gamma$.)

63. INTERPRET We identify our system as the horizontal piston-cylinder system containing an ideal gas. We want to demonstrate that the piston undergoes simple harmonic motion when slightly displaced.

DEVELOP Since the piston-cylinder system is horizontal, we do not need to consider the force of gravity on the piston. At equilibrium, the pressure forces from inside and outside the piston are equal, so the gas pressure at the equilibrium position of the piston is p_0 . We also assume that the gas temperature at equilibrium is T_0 , so $p_0 V_0 = nRT_0$, where $V_0 = Ax_0$ is the volume at equilibrium. When the piston is displaced from its equilibrium position by an amount Δx (positive to the right), the horizontal force on it is $F = pA - p_0A = (p - p_0)A$, and Newton's second law gives an acceleration of

$$\frac{d^2(\Delta x)}{dt^2} = \frac{(p - p_0)A}{M}$$

For isothermal expansions and compressions of the gas,

$$pV = p_0V_0 \rightarrow pA(x_0 + \Delta x) = p_0Ax_0$$

For small displacements $\Delta x \ll x_0$, the pressure is

$$p = p_0 \frac{x_0}{x_0 + \Delta x} = p_0 \frac{1}{1 + (\Delta x/x_0)} \approx p_0 [1 - (\Delta x/x_0)]$$

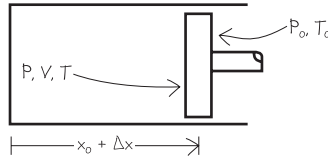
(see the binomial approximation in Appendix A). Substituting the expression for p into Newton's second law equation allows us to show that the piston executes simple harmonic motion.

EVALUATE Combining the two equations yields

$$\frac{d^2(\Delta x)}{dt^2} = \frac{(p - p_0)A}{M} = -\frac{p_0A}{M} \frac{\Delta x}{x_0} = -\omega^2 \Delta x$$

where $\omega^2 = p_0A/Mx_0$. This is the equation for simple harmonic motion of the piston, about its equilibrium position x_0 , with angular frequency $\omega = \sqrt{p_0A/Mx_0}$. Since $p_0V_0 = p_0Ax_0 = nRT_0$, we may eliminate x_0 to obtain

$$\omega = \frac{p_0A}{\sqrt{MnRT_0}}$$



ASSESS In order for the gas temperature to remain constant, as assumed above, heat must flow into and out of the gas. This requires time, so the motion of the piston must be very slow. If the motion is rapid (or if the cylinder is thermally insulated), there is no time for heat transfer in the gas and the expansions and compressions are adiabatic. In this case,

$$pV^\gamma = p_0V_0^\gamma \rightarrow pA^\gamma(x_0 + \Delta x)^\gamma = p_0A^\gamma x_0^\gamma$$

or $p = p_0(1 + \Delta x/x_0)^{-\gamma}$. For small displacements $\Delta x \ll x_0$, we have $p \approx p_0(1 - \gamma\Delta x/x_0)$ and

$$\frac{d^2(\Delta x)}{dt^2} = \frac{(p - p_0)A}{M} = -\frac{\gamma p_0A}{M} \frac{\Delta x}{x_0} = -\omega^2 \Delta x$$

where $\omega^2 = \gamma p_0A/Mx_0$. This represents simple harmonic motion with

$$\omega = \sqrt{\gamma p_0A/Mx_0} = p_0A \sqrt{\gamma/MnRT_0}$$

64. (a) The circular path is most easily described by dimensionless variables:

$$x = (V - \bar{V})/r_v, \quad \text{and} \quad y = (P - \bar{P})/r_p,$$

where $\bar{V} = \frac{1}{2}(V_{\min} + V_{\max})$ and $\bar{P} = \frac{1}{2}(P_{\min} + P_{\max})$, and $r_v = \frac{1}{2}(V_{\max} - V_{\min})$ and $r_p = \frac{1}{2}(P_{\max} - P_{\min})$. (The “center” of the circle is (\bar{V}, \bar{P}) , and the “radius,” in volume and pressure units, is r_v or r_p , respectively.) Therefore, $x^2 + y^2 = 1$ for the path.

The work done in one clockwise cycle is (see Equations 7.1 or 19.5)

$$W = \oint P dV = \oint (r_p y + \bar{P}) r_v dx = r_p r_v \oint y dx + r_v \bar{P} \oint dx,$$

The first integral is the area of the unit circle, $x^2 + y^2 = 1$, which is π , and the second integral is zero. Therefore,

$$W = r_p r_v \pi = \frac{1}{4} \pi (P_{\max} - P_{\min})(V_{\max} - V_{\min}) = \frac{1}{4} \pi (550 - 150) \text{ kPa} (11 - 3) \text{ L} = 2.51 \text{ kJ.}$$

(For those unfamiliar with integrals over closed paths, they are the difference between the integrals over the upper and lower parts, as

explained in Fig. 18.13. In this case, $\oint y dx = \int_{-1}^1 y_+(x) dx - \int_{-1}^1 y_-(x) dx$, where $y_{\pm}(x) = \pm \sqrt{1 - x^2}$ are the upper and lower semicircles. Then $\oint y dx = 2 \int_{-1}^1 \sqrt{1 - x^2} dx = \pi$. The integral of a constant over a closed path is zero, since the upper and lower parts are the same. Then $\oint dx = \int_{-1}^1 dx - \int_{-1}^1 dx = 0$.)

(b) The maximum temperature reached is that of the isotherm ($PV = \text{constant}$) which is tangent to the upper semicircle in the PV diagram. In terms of the dimensionless variables, the equation of an isotherm is

$xy + (\bar{P}/r_p)x + (\bar{V}/r_v)y = \text{constant}$, where $y = y_+(x) = \sqrt{1 - x^2}$ on the upper semicircle. For the particular numerical values given, $(\bar{P}/r_p = V/r_v = \frac{7}{4})$, the equation simplifies to $xy + \frac{7}{4}(x + y) = \text{constant}$. The condition for a maximum is:

$$0 = \frac{d}{dx} \left[xy + \frac{7}{4}(x + y) \right] = y - \frac{x^2}{y} + \frac{7}{4} \left(1 - \frac{x}{y} \right) = \left(1 - \frac{x}{y} \right) \left(x + y + \frac{7}{4} \right)$$

where we used $dy/dx = d(\sqrt{1 - x^2})/dx = -x/\sqrt{1 - x^2} = -x/y$. The second factor, $x + y + \frac{7}{4}$, is never zero on the unit circle, so the maximum occurs for $1 - x/y = 0$, or $x_m = y_m = \sqrt{1 - x_m^2} = 1/\sqrt{2}$. The corresponding values of volume and pressure are:

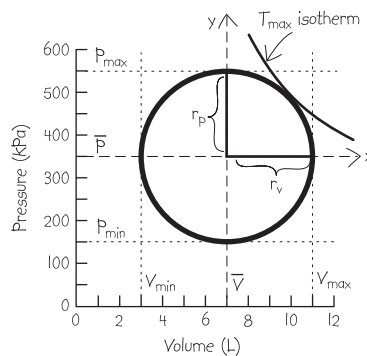
$$V_m = r_v x_m + \bar{V} = \frac{1}{\sqrt{2}} \left(\frac{11 - 3}{2} \right) \text{ L} + \left(\frac{11 + 3}{2} \right) \text{ L} = 9.83 \text{ L},$$

and

$$P_m = r_p y_m + \bar{P} = \frac{1}{\sqrt{2}} \left(\frac{550 - 150}{2} \right) \text{ kPa} + \left(\frac{550 + 150}{2} \right) \text{ kPa} = 491 \text{ kPa}$$

The maximum temperature follows from the ideal gas law:

$$T_m = \frac{P_m V_m}{nR} = \frac{(491 \text{ kPa})(9.83 \text{ L})}{1.3(8.314 \text{ J/K})} = 447 \text{ K}$$



65. INTERPRET We identify the thermodynamic process as adiabatic with $Q = 0$.

DEVELOP In an adiabatic process, the temperature and pressure are related by Equation 18.11a: $pV^\gamma = \text{constant}$.

The equation can be rewritten as

$$pV^\gamma = p^{1-\gamma}(pV)^\gamma = p^{1-\gamma}(nRT)^\gamma$$

which implies that $p^{1-\gamma}T^\gamma = \text{constant}$. The work done by the air is

$$W = \frac{p_i V_i - p_f V_f}{\gamma - 1}$$

EVALUATE (a) Using the equation above, the final temperature is

$$T_f = T_i \left(\frac{p_f}{p_i} \right)^{(\gamma-1)/\gamma} = (262 \text{ K}) \left(\frac{86.5 \text{ kPa}}{62.0 \text{ kPa}} \right)^{(1.4-1)/1.4} = 288 \text{ K} = 15.2 \text{ C}$$

(b) The work done on one cubic meter of air (the negative of Equation 18.12) is

$$\begin{aligned} W_{\text{on gas}} &= -\frac{p_i V_i - p_f V_f}{\gamma - 1} = \frac{(p_f V_f - p_i V_i)}{\gamma - 1} = \frac{p_i V_i}{\gamma - 1} \left[\left(\frac{p_f}{p_i} \right)^{(\gamma-1)/\gamma} - 1 \right] \\ &= \frac{(62.0 \text{ kPa})(1 \text{ m}^3)}{1.4 - 1} \left[\left(\frac{86.5 \text{ kPa}}{62.0 \text{ kPa}} \right)^{(1.4-1)/1.4} - 1 \right] = 15.5 \text{ kJ} \end{aligned}$$

where we used Equation 18.11a to eliminate V_f .

ASSESS The final volume is

$$V_f = V_i \left(\frac{p_i}{p_f} \right)^{1/\gamma} = (1 \text{ m}^3) \left(\frac{62.0 \text{ kPa}}{86.5 \text{ kPa}} \right)^{1/1.4} = 0.788 \text{ m}^3$$

which is less than the initial volume V_i . So, positive work is done on the air to compress it.

66. INTERPRET We derive the relationship between temperature and volume for adiabatic processes.

DEVELOP The relationship between pressure and volume is $pV^\gamma = \text{constant}$. We can use this, and the ideal gas law $pV = nRT$, to derive equation 18.11b, $TV^{\gamma-1} = \text{constant}$.

EVALUATE $pV = nRT \rightarrow p = n \frac{RT}{V}$. We substitute this into $pV^\gamma = \text{constant}$ to obtain $(n \frac{RT}{V})V^\gamma = (nR)TV^{\gamma-1} = \text{constant}$. We note that $\frac{\text{constant}}{nR}$ is still a constant, albeit a different one, so $TV^{\gamma-1} = \text{constant}$.

ASSESS We have shown what was required.

67. INTERPRET The problem involves a cyclic process, and we identify four separate stages of the cycle: adiabatic compression, isobaric expansion, adiabatic expansion, and isochoric cooling.

DEVELOP From Table 18.1, the work done and heat absorbed during each of the four processes comprising the diesel cycle is:

$$\begin{aligned} 1 \rightarrow 2 \text{ (adiabatic)} & \quad W_{12} = (p_1 V_1 - p_2 V_2)/(\gamma - 1), \quad Q_{12} = 0 \\ 2 \rightarrow 3 \text{ (isobaric)} & \quad W_{23} = p_2 (V_3 - V_2), \quad Q_{23} = nC_p (T_3 - T_2) \equiv Q_h \\ 3 \rightarrow 4 \text{ (adiabatic)} & \quad W_{34} = (p_3 V_3 - p_4 V_4)/(\gamma - 1), \quad Q_{34} = 0 \\ 4 \rightarrow 1 \text{ (isochoric)} & \quad W_{41} = 0, \quad Q_{41} = nC_v (T_1 - T_4) \equiv -Q_c \end{aligned}$$

For the whole cycle, the work done is

$$W = \frac{p_1 V_1 - p_2 V_2 + p_3 V_3 - p_4 V_4}{(\gamma - 1)} + p_3 V_3 - p_2 V_2 = \frac{\gamma(p_3 V_3 - p_2 V_2) - p_4 V_4 + p_1 V_1}{(\gamma - 1)}$$

while the heat added can be written as

$$Q_h = nC_p (T_3 - T_2) = \frac{\gamma(p_3 V_3 - p_2 V_2)}{\gamma - 1}$$

where we used the ideal gas law, $nT = PV/R$, and

$$\frac{C_p}{R} = \frac{C_p}{C_p - C_v} = \frac{\gamma}{\gamma - 1}$$

Therefore, the work done by the system can be rewritten as

$$W = Q_h - \frac{(P_4V_4 - P_1V_1)}{(\gamma - 1)} = Q_h - \frac{(P_4V_4 - P_1V_1)Q_h}{\gamma(P_3V_3 - P_2V_2)}$$

The efficiency is

$$e = \frac{W}{Q_h} = 1 - \frac{(P_4V_4 - P_1V_1)}{\gamma(P_3V_3 - P_2V_2)}$$

EVALUATE The adiabatic law can now be used to express every product in terms of P_2V_2 and the compression and cutoff ratios: $P_2 = P_3$, $V_1 = V_4$, $V_1/V_2 = r$, $V_3/V_2 = r_c$, $P_1V_1^\gamma = P_2V_2^\gamma$, and $P_3V_3^\gamma = P_4V_4^\gamma$, so

$$\begin{aligned} p_1V_1 &= p_2V_2r^{1-\gamma} \\ p_3V_3 &= p_2V_2r_c \\ p_4V_4 &= p_2V_2r^{1-\gamma}r_c^\gamma \end{aligned}$$

Using these expressions, we find the efficiency to be

$$e = 1 - \frac{r^{1-\gamma}(r_c^\gamma - 1)}{\gamma(r_c - 1)}$$

ASSESS The efficiency of the Diesel engine is always less than one. Diesel engines are more efficient than gasoline engines.

68. INTERPRET We find the work done during a reversible process in which we are given an equation for pressure.

DEVELOP The most general equation for work done by a gas is $W = \int_{V_1}^{V_2} P dV$. We are given the initial pressure $P_0 = 101.3 \text{ kPa}$, the initial volume $V_0 = 17 \text{ m}^3$, and an equation relating pressure and volume: $(\frac{P}{P_0})^2 = \frac{V}{V_0}$. The final pressure is given as $P_f = 1.4 \text{ atm} = 141.8 \text{ kPa}$.

We will need to solve the given equation for P , find the final volume, and integrate.

EVALUATE The integrand is given by $(\frac{P}{P_0})^2 = \frac{V}{V_0} \rightarrow P = P_0\sqrt{\frac{V}{V_0}}$. The final volume is $V_f = V_0(\frac{P_f}{P_0})^2$. We integrate to find the work:

$$\begin{aligned} W &= \int_{V_0}^{V_f} P_0\sqrt{\frac{V}{V_0}} dV = \frac{P_0}{\sqrt{V_0}} \frac{2}{3} V_0^{\frac{3}{2}} \left[\left(\frac{P_f}{P_0} \right)^2 \right] = \frac{2P_0}{3\sqrt{V_0}} \left[V_0^{\frac{3}{2}} \left(\frac{P_f}{P_0} \right)^3 - V_0^{\frac{3}{2}} \right] \\ \rightarrow W &= \frac{2P_0V_0}{3} \left[\left(\frac{P_f}{P_0} \right)^3 - 1 \right] = 2.0 \text{ MJ} \end{aligned}$$

ASSESS The question says the gas is “compressed” to the new pressure, but the equation given has pressure increase with volume. If we look closely at the behavior of the p-V graph for this problem, we see that the work done is positive and the temperature must increase.

69. INTERPRET We derive the equivalent of $W = nRT \ln(\frac{V_2}{V_1})$ for a non-ideal gas that includes Van der Waals forces. We are given the equation relating pressure and volume for a Van der Waals gas.

DEVELOP The equation relating pressure and volume for a Van der Waals gas is given as $[p + a(\frac{n}{V})^2](V - nb) = nRT$. We solve this for pressure, and integrate to find the work.

EVALUATE

$$\begin{aligned} \left[p + a\left(\frac{n}{V}\right)^2 \right] (V - nb) &= nRT \rightarrow p(V - nb) + a\left(\frac{n}{V}\right)^2 (V - nb) = nRT \\ \rightarrow p &= \frac{1}{V - nb} \left[nRT - a\left(\frac{n}{V}\right)^2 (V - nb) \right] = \frac{nRT}{V - nb} - a\left(\frac{n}{V}\right)^2 \end{aligned}$$

Integrate this to find work:

$$W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \left[\frac{nRT}{V-nb} - a \left(\frac{n}{V} \right)^2 \right] dV = \left[nRT \ln(V-nb) + \frac{an^2}{V} \right]_{V_1}^{V_2}$$

$$\rightarrow W = nRT \ln \left(\frac{V_2-nb}{V_1-nb} \right) + an^2 \left(\frac{1}{V_2} - \frac{1}{V_1} \right)$$

ASSESS This simplifies to $W = nRT \ln \left(\frac{V_2}{V_1} \right)$ if a and b are zero.

- 70. INTERPRET** An ideal gas expands along a given path, and we need to find the work done by the gas. We can do this by integrating PdV .

DEVELOP The gas goes from (p_1, V_1) to (p_2, V_2) , where $p_2 = 2p_1$ and $V_2 = 2V_1$. The path it takes is along $p = p_1 \left[1 + \frac{(V-V_1)^2}{V_1^2} \right]$. The work is $W = \int_{V_1}^{V_2} P dV$, so we simply integrate.

EVALUATE

$$W = \int_{V_1}^{V_2} P dV = \int_{V_1}^{2V_1} p_1 \left[1 + \frac{(V-V_1)^2}{V_1^2} \right] dV = \frac{p_1}{V_1^2} \int_{V_1}^{2V_1} [V^2 - 2V_1V + 2V_1^2] dV$$

$$\rightarrow W = \frac{p_1}{V_1^2} \left[\frac{1}{3}V^3 - V_1V^2 + 2V_1^2V \right]_{V_1}^{2V_1} = \frac{p_1}{V_1^2} \left[\frac{1}{3}(8V_1^3 - V_1^3) - V_1(4V_1^2 - V_1^2) + 2V_1^2(2V_1 - V_1) \right]$$

$$\rightarrow W = p_1V_1 \left[\frac{7}{3} - 3 + 2 \right] = \frac{4}{3}p_1V_1$$

ASSESS The units are pressure times volume, which we know is correct.

- 71. INTERPRET** We calculate the rate $\frac{dT}{dy}$ at which air cools as it rises, approximating the process as adiabatic.

DEVELOP The hydrostatic equation is $\frac{dp}{dh} = \rho g = -\frac{dp}{dy}$, and for adiabatic processes $nC_v dT = -pdV$. We differentiate the ideal gas law $pV = nRT$, with the previous substitution, to find the relationship between dT and dp . Finally we use the hydrostatic equation to relate dp to dy and thus find $\frac{dT}{dy}$. The molecular weight of air is given as

$M = 0.029 \text{ kg/mol}$, and $\gamma = 1.4$.

EVALUATE

$$pV = nRT \rightarrow nRdT = pdV + Vdp = -nC_v dT + Vdp = nRdT$$

$$\rightarrow Vdp = n(R + C_v)dT = nC_p dT$$

Volume is $V = \frac{m}{\rho} = \frac{nM}{\rho}$, so $\frac{M}{\rho} dp = C_p dT$. From $\rho g = -\frac{dp}{dy}$, we have $dp = -\rho g dy$, so $\frac{M}{\rho} (-\rho g dy) = C_p dT \rightarrow \frac{dT}{dy} = -\frac{Mg}{C_p}$.

$$C_p = \frac{\gamma R}{\gamma - 1}, \text{ so } \frac{dT}{dy} = -\frac{Mg}{\gamma R} (\gamma - 1) = -0.0098 \text{ K/m}$$

ASSESS This value, -9.8 K/km , is the approximate change in temperature as you gain altitude.

- 72. INTERPRET** We calculate the efficiency and waste heat of a power plant, given the heat input and the power produced.

DEVELOP The efficiency is $e = \frac{\text{power out}}{\text{power in}}$. The waste heat is the difference between the power in and the power out. We need to know that the heat is wasted at a rate of less than 400 MW, and the power plant has efficiency $e \geq 50\%$.

EVALUATE $e = \frac{360 \text{ MW}}{670 \text{ MW}} = 54\%$. Heat is lost at a rate of $P_{\text{lost}} = 670 \text{ MW} - 360 \text{ MW} = 310 \text{ MW}$.

ASSESS This power plant meets the requirements.

- 73. INTERPRET** We find the pressure and volume of air within a diving bell, given that the temperature varies in such a way that the pressure and volume vary according to a given equation.

DEVELOP The equation relating pressure and temperature is given as $p = p_0 \sqrt{\frac{V_0}{V}}$, where $V_0 = 17 \text{ m}^3$ and $p_0 = 1 \text{ atm}$. The value of $p_{\text{max}} = 1.5 \text{ atm}$. We can plug these values into the equation, and see what is the resulting value of V . If it's more than $V_{\text{min}} = 8.66 \text{ m}^3$, the design is ok.

EVALUATE Solve the given equation for V : $p = p_0 \sqrt{\frac{V_0}{V}} \rightarrow V = \frac{p_0^2 V_0}{p^2} = 7.56 \text{ m}^3$.

ASSESS The design needs work. Note also that since the pressure appears only in a ratio, we do not need to convert to SI units.

- 74. INTERPRET** We are given a temperature change and a volume change, and wish to find the value of γ . The process is adiabatic.

DEVELOP We can use $TV^{\gamma-1} = \text{constant}$. We are given that $V_f = 0.177V_i$ and $T_f = 2T_i$

EVALUATE

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1} = 2T_i (0.177V_i)^{\gamma-1} = T_i V_i^{\gamma-1} (2(0.177)^{\gamma-1})$$

$$\rightarrow 1 = 2(0.177)^{\gamma-1} \rightarrow \ln\left(\frac{1}{2}\right) = (\gamma-1)\ln(0.177)$$

$$\rightarrow \gamma = \frac{\ln\left(\frac{1}{2}\right) + \ln(0.177)}{\ln(0.177)} = 1.40$$

ASSESS This is the expected value for a diatomic gas.

- 75. INTERPRET** We find the heat required to heat a volume of gas at constant volume.

DEVELOP The molar specific heat of air at this range of temperatures is $C_v = 2.5R$. The temperature change is from 0°C to 20°C , and the initial volume at 0°C is 5.0 L. The heat required will be $Q = nC_v\Delta T$, so if we find the number of moles n using $pV = nRT$, we can find Q . We will assume that the initial pressure is $p_i = 101.3 \text{ kPa}$.

EVALUATE

$$pV = nRT \rightarrow n = \frac{pV}{RT}. \quad Q = nC_v\Delta T = \frac{p_i V_i}{R T_i} (2.5R) \Delta T = 93 \text{ J}$$

ASSESS Note that we are using the ideal-gas law, so it is important to use absolute temperature and volume in m^3 .