

Answers

Measurements and Uncertainties

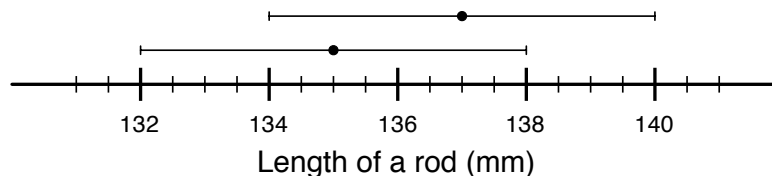
1.

- a) 5.03 ± 0.04 m
- b) Applying the rules as seen yields 2 ± 1 s, but here rounding up changes the result by a very large fraction of the initial value, therefore 1.5 ± 1 s is considered a better answer.
- c) $(-3.2 \pm 0.3) \times 10^{-19}$ C
- d) $(5.6 \pm 0.7) \times 10^{-7}$ m
- e) $(3.27 \pm 0.04) \times 10^3$ g cm /s

2.

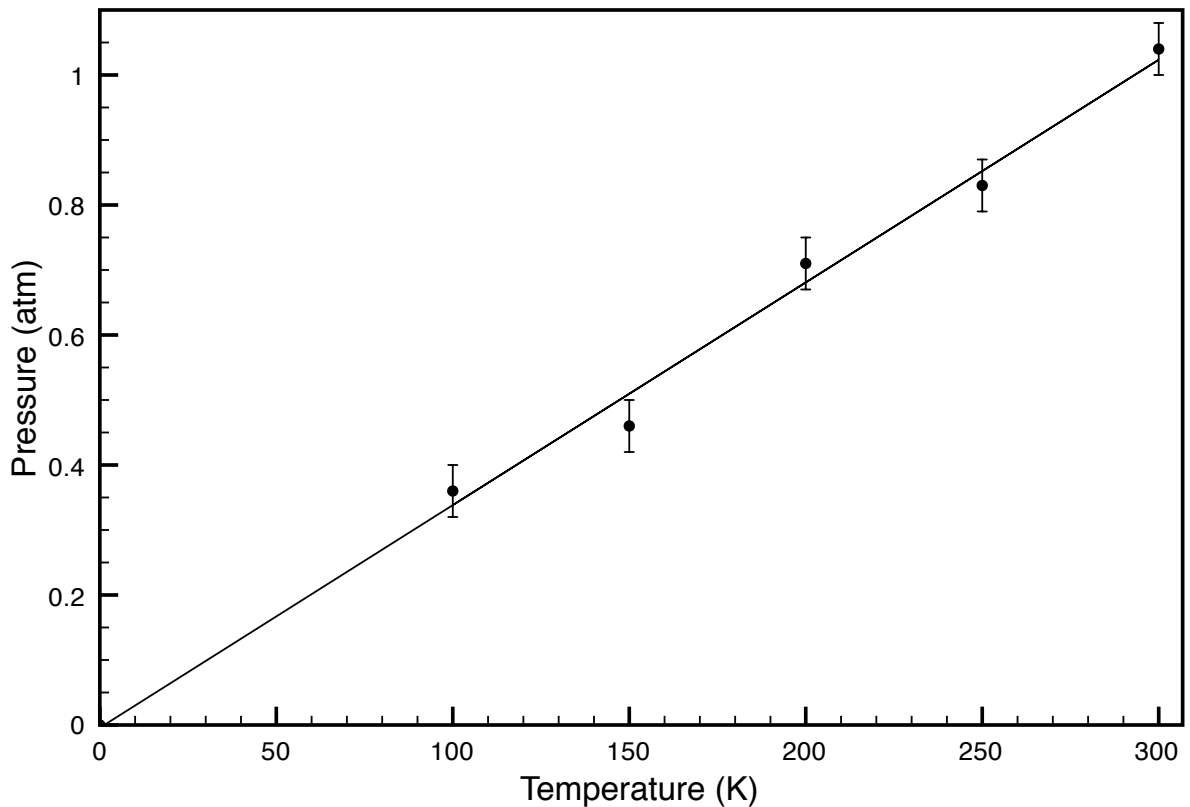
- a) $x = 3 \pm 1$ mm
- b) $t = (1.23 \pm 0.05) \times 10^6$ s
- c) $\lambda = (5.33 \pm 0.03) \times 10^{-7}$ m
- d) $r = (5.4 \pm 0.3) \times 10^{-7}$ mm

3. The discrepancy is 2 mm but it is not significant because it is within the uncertainty range of both measurement. We can also say that the uncertainties overlap, therefore the two values are in agreement.

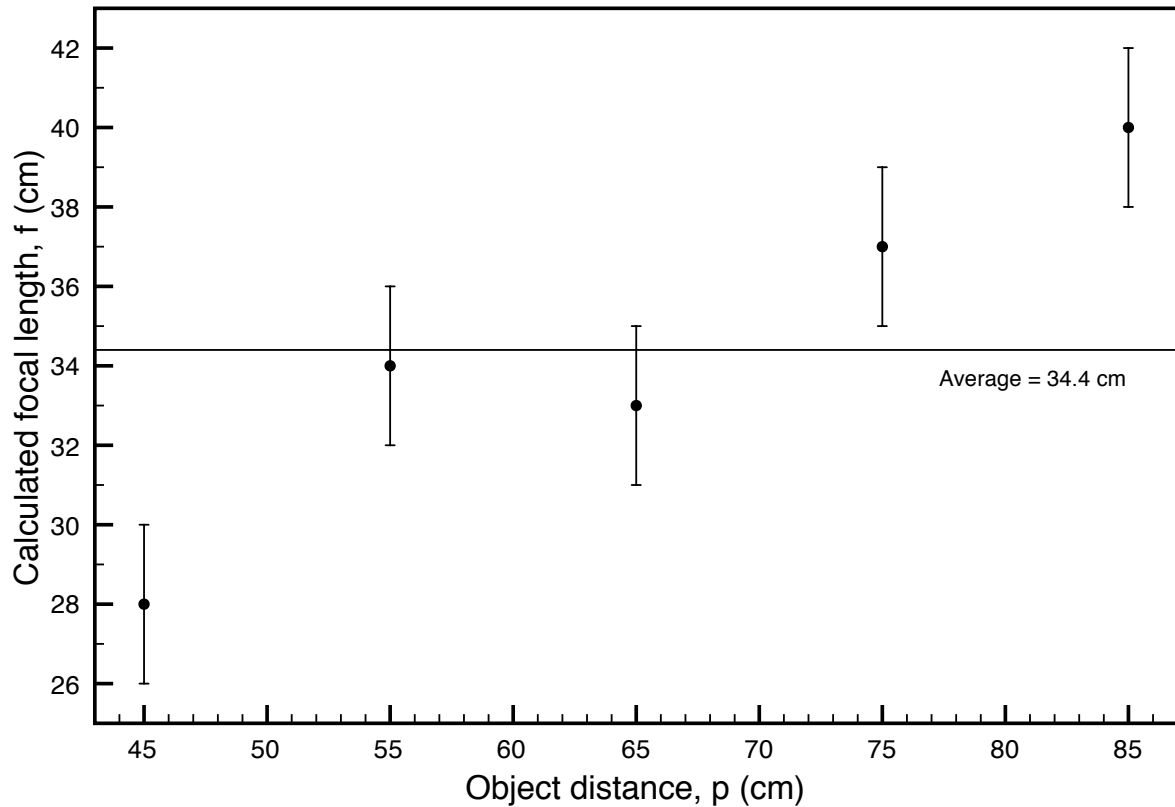


4. The best estimate is the average, here 1.9 g/cm^3 . The difficulty is to estimate the uncertainty on that best estimate.
- The simplest way, which will also yield the worst evaluation of the uncertainty is to calculate $(\text{max}-\text{min})/2$, which would give $1.9 \pm 0.1 \text{ g/cm}^3$. This however overestimate the uncertainties, because an average tends to reduce random errors.
 - A much better method is to realize that the more values one takes, the smaller the uncertainties will be, since the average will tend to reduce random errors. Therefore one could use as an uncertainty range $(\text{max}-\text{min})/n$, where n is the number of values. Using this method when $n < 10$ values, will yield a better result than the previous method and the result would be $1.90 \pm 0.04 \text{ g/cm}^3$. For $n > 10$ values, this method will underestimate the uncertainties on the average.
 - Finally the best method would be to calculate the *standard deviation of the mean*, which here would also yield $1.90 \pm 0.04 \text{ g/cm}^3$. Certainly for repeated measurements more than 10 times, this is the best method.
5. $l = 83.5 \pm 0.1$ cm. Remember to keep a single significant figure in the uncertainty.

6. In all cases but one, the measured value of $L - L'$ is smaller than the uncertainty. In the one exceptional case (-2.2 ± 2), it is only slightly larger. Therefore, the observed values are consistent with the expected value zero.
7. As seen in the table below, the measured value is always significantly smaller than the expected value, indicating a systematic error. This error is possibly due to a systematic error in the measurement of the angle, or more probably the friction force acting on the object was not negligible, thus creating an acceleration smaller than expected.
8. The straight line shown in the figure below passes through the origin and through, or close to, all error bars. Therefore, the data are consistent with P being proportional to T .



9. The calculated focal length should be a constant value. Plotting f against p as shown below with the correct error bars, shows however that most values disagree with one another, and that their uncertainty range do not include their average value. One must then conclude that according to the data, this lens does not have a unique focal length.



10.

- a) $x = 540 \pm 20$ m
- b) $v = 66 \pm 5$ m/s
- c) $\lambda = (670 \pm 30) \times 10^{-9}$ m

11. It's uncertainty is 0.03%

12. It's uncertainty is 0.2%

13. $x = 6.1 \pm 0.1$;
 $y = 1.12 \pm 0.02$;
 $z = 9.1 \pm 0.2$

14. $q = 2720$ Ns $\pm 10\%$ or $q = 2700 \pm 300$ Ns.

15.

- a) 3 ± 7
- b) 40 ± 20
- c) 0.5 ± 0.1
- d) 300 ± 20

16.

- a) 35 ± 4 cm
- b) 11 ± 4 cm
- c) 36 ± 9 cm s
- d) 110 ± 40 g cm / s

17.

- a) $T = 0.48 \pm 0.02$ s or 4%
- b) $T = 0.470 \pm 0.005$ s or 1%
- c) No, because of other factors such as the air resistance, which will affect the amplitude of oscillations of the pendulum and ultimately stop it.

18. The volume of a sphere is $v = \frac{4}{3}\pi r^3$. therefore, applying our simple rules one finds $v = 34 \pm 5$ m³.