

Problem Session

Rotational Kinematics

1. The turntable of an old record player has a diameter of 30 cm. It starts from rest and takes 2 s to reach its rotation speed, which is $33 \frac{1}{3}$ rev/min.
Determine:
 - a) The angular acceleration.
 - b) The number of revolutions in 5 s.
 - c) The time necessary to make the 2 first revolutions.
 - d) The magnitude of the radial and tangential accelerations of a point on the circumference of the turntable at the instant $t = 1$ s.
 - e) same thing as d) but for the instant $t = 3$ s.

2. An old record of 30 cm diameter turns at a regular speed of $33 \frac{1}{3}$ rev/min for 20 min.
Determine the linear speed...
 - a) of a point at the beginning of the recording, for which $r = 14.5$ cm.
 - b) of a point at the end of the recording for which $r = 6.2$ cm.

3. The angular position on a disc of radius $r = 6$ cm is given by $\theta = 10 - 5t + 4t^2$, where t is expressed in seconds and θ in radians. Determine:
 - a) the average angular speed between 1 and 3 s.
 - b) the linear speed of a point on the circumference at $t = 2$ s
 - c) the magnitude of the radial and tangential accelerations of a point on the circumference at $t = 2$ s.

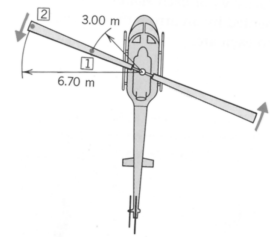
4. A particle follows a circular path of radius r with an angular speed ω and an angular acceleration α . Show that the linear acceleration of the particle has for magnitude:

$$a = r\sqrt{\omega^4 + \alpha^2}$$

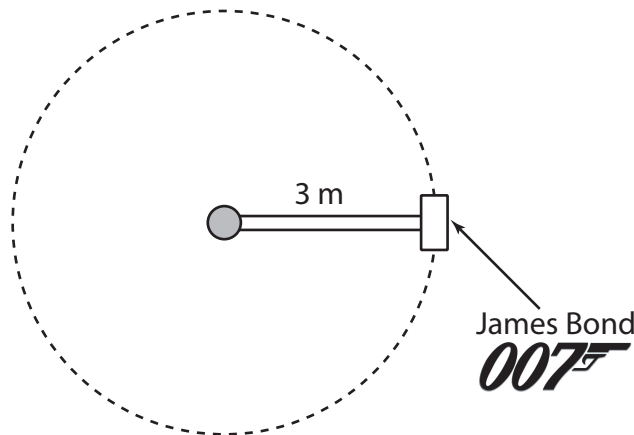
5. At $t = 0$, a wheel turns at 50 rev/min. A motor gives it a constant angular acceleration of 0.5 rad/s^2 until it reaches 100 rev/min. The motor is then disconnected from the wheel. How many turns did the wheel make at $t = 20$ s?

6. The wheel of an automobile has a radius of 20 cm. It turns initially at 120 rev/min, but slows down at a constant rate. During the following minute it makes 90 revolutions, but has not yet stopped.
 - a) What is the angular acceleration of the wheel?
 - b) What is the remaining distance the car will cover before stopping. Consider that the wheel is not slipping.

7. A helicopter blade starts from rest and, with a constant angular acceleration, reaches an angular speed of 6.50 rev/s in 5.00 s. At any point on a blade, how much time passes before the magnitude of the tangential acceleration equals the magnitude of the centripetal acceleration?



8. A “*centrifuge*” is an astronaut training vehicle used to prepare astronauts to the large acceleration they will face during takeoff. It is a small single seater capsule, which is spun quickly in a circle by a large motorized arm. The acceleration of the astronaut is determined by changing the angular speed of the capsule. An acceleration of 3g ($g = 9.8 \text{ m/s}^2$, the acceleration due to gravity at the surface of our planet), is considered a safe amount, 7g is enough to make most people pass out, and 20g would be fatal.

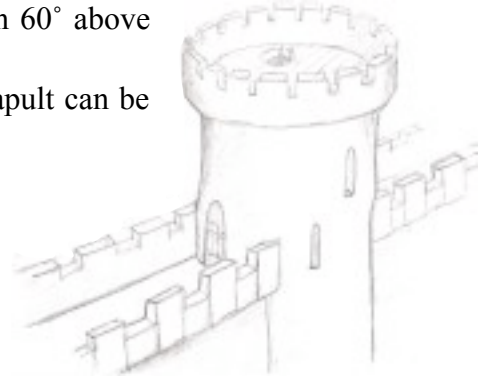
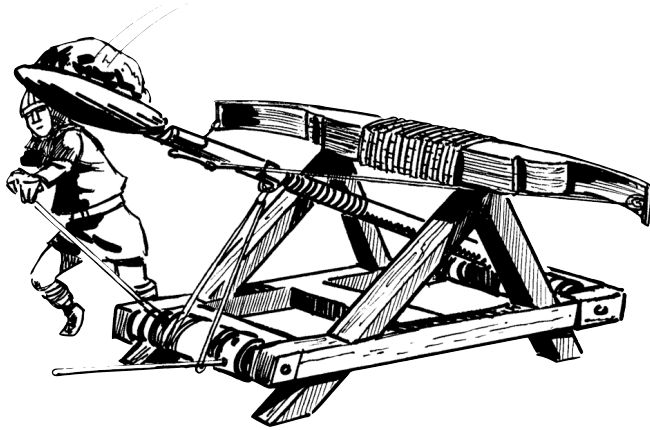


- James Bond**, in the movie *Moonraker*, was given a personal demonstration of Sir Hugo Drax’s centrifuge, the acceleration was going to be 3g. What should be the angular speed of the capsule under such conditions?
- If it took 30 s to reach this angular speed, what was the angular acceleration of the capsule during that time.
- Unfortunately for Bond, after only 15 s of operation at 3g, Drax's henchman, Chang took over the centrifuge and disabled the panic button, increasing the machine to a suicidal 12g, in another 30 s. What was the angular acceleration of the capsule during that time?
- As a last moment attempt, Bond managed to fire his wrist dart gun into the control panel of the capsule and stop the machine, which comes to rest in 60 s. How many revolutions did James Bond do while in the capsule?

9. ***This problem involves both rotational kinematics and 2D projectile motion.***

A catapult is a medieval siege engine which uses an arm to hurl projectiles at a great distance. The particular catapult shown in the figure below, has an arm 5 m long attached by ropes to a large wooden bow that will make the arm move along a circular path when the catapult is released. Assume the catapult's arm starts in an horizontal position and is accelerated uniformly for 0.25 s to reach a position 60° above the horizontal, at which point a 20 kg rock is released.

Calculate the minimum and maximum distance, this catapult can be placed in order to throw the rock above a 10 m wall.



Answers

1. a) 1.75 rad/s^2 b) 2.22 rev c) 4.60 s d) $a_t = 0.263 \text{ m/s}^2, a_c = 0.549 \text{ m/s}^2$
e) $a_t = 0 \text{ m/s}^2, a_c = 1.83 \text{ m/s}^2$
2. a) 0.506 m/s b) 0.216 m/s
3. a) $\bar{\omega} = 11.0 \text{ rad/s}$ b) $v = 0.660 \text{ m/s}$ c) $a_t = 0.480 \text{ m/s}^2, a_c = 7.26 \text{ m/s}^2$
5. 29.0 revolutions
6. a) $\alpha = -0.105 \text{ rad/s}^2$ b) 37.7 m
7. 0.350 s
8. a) 3.13 rad/s b) 0.104 rad/s^2 c) 0.104 rad/s^2 d) 59.8 revolutions (from start to finish).
9. $x_{\min} = 10.54 \text{ m}, x_{\max} = 144.5 \text{ m}$