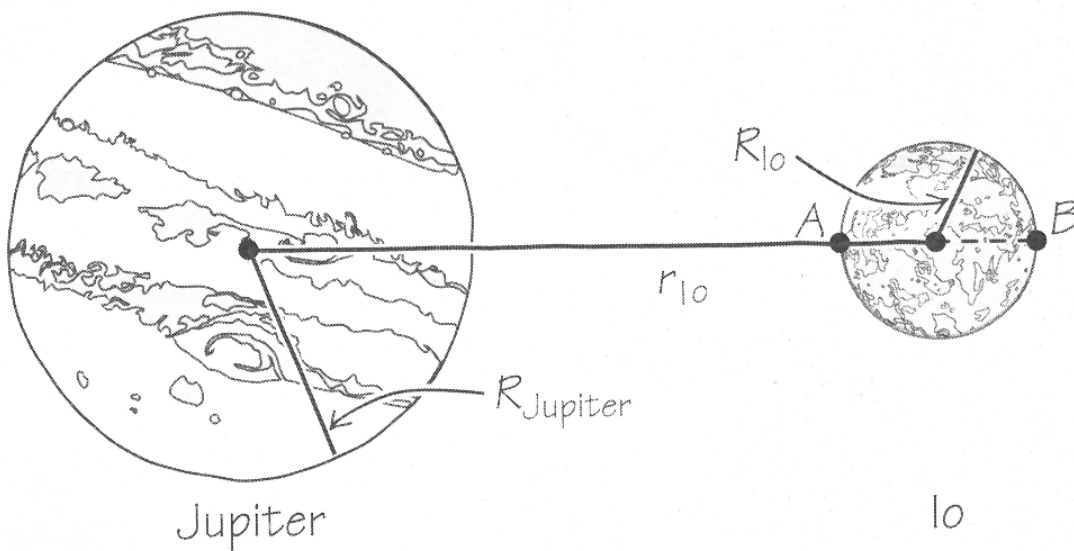


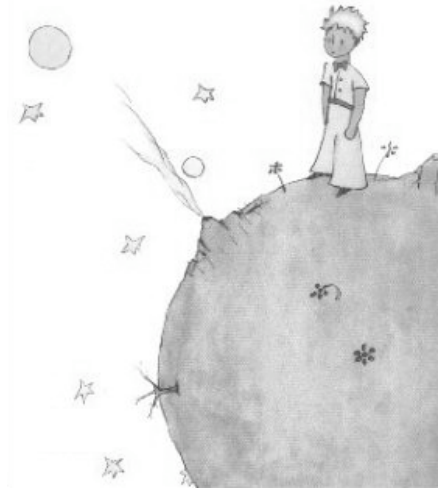
Problem Session

Gravitation and Satellite Motion

1. Jupiter's moon Io has a radius of 1815 km, a mass of 4.7×10^{-5} that of Jupiter, and an orbital radius about Jupiter that is 5.95 times Jupiter's own radius. Given that Jupiter's mass is 318 times that of the Earth and that its radius is 26 times larger than Earth's, calculate the acceleration due to gravity on Io at the point nearest Jupiter (A) and the point farthest from Jupiter (B). $M_{Earth} = 5.976 \times 10^{24}$ kg, $R_{Earth} = 6.374 \times 10^6$ m.



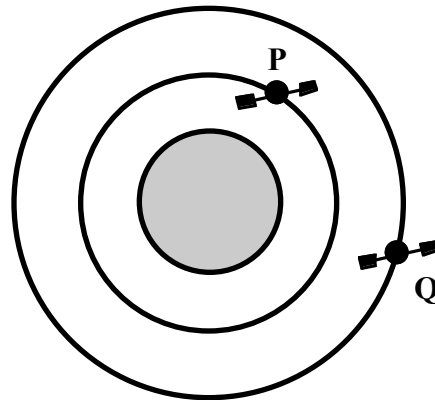
2. The Little Prince (a character in a book by Antoine de Saint-Exupéry) lives on the spherically symmetric asteroid B-612. The density of asteroids, including B-612, is $5.2 \times 10^3 \text{ kg/m}^3$. Assume that the asteroid does not rotate. The Little Prince noticed that he felt lighter whenever he walked quickly around his asteroid. In fact, he found that he felt weightless and started to orbit the asteroid like a satellite just above the surface whenever he speeded up to 2 m/s.
- a) Estimate the radius of the asteroid from these data.



3. The mass of the planet Mars is $6.42 \times 10^{23} \text{ kg}$, its radius is 3393 km, and a solar day on Mars lasts 24h, 39 min and 35s.
- a) What would be the weight of a 500 kg robot sent on this planet?
- b) Calculate the period (in hours) of a satellite in circular orbit, 95 km above the surface of Mars.
- c) At which altitude above the martian surface, must one place a satellite so it follows a synchronous¹ orbit?

¹ For Mars we say **areosynchronous**, and for Earth **geosynchronous**. *Do you know why?*

4. Two identical satellites P and Q are in circular orbits around Earth. The orbital radius of P is 1.5 times smaller than the one of Q . What is the ratio of their rotation period (T_P/T_Q)?



5. You are working with an international team of scientists trying to send a robot on Titan, the largest of Saturn's satellites. The mass of Titan is 13.5×10^{22} kg, its radius is 2575 km and rotates about its axis in 15.945 days².
- What would be the weight of a 500 kg robot sent on Titan?
 - At which altitude above Titan's surface, must your team place a satellite so it follows a synchronous orbit and remains in constant contact with the robot on the surface?
 - The rotation of Titan is itself synchronous, meaning that it takes the same time to rotate on itself and around Saturn, thus always facing Saturn the same way³. If Titan is observed to have an orbital radius of 1222×10^3 km, Calculate the mass of Saturn.

² The rotation period was determined through the comparison of data taken by Voyager 1 (1980) and Hubble Space Telescope (1994).

³ Our Moon, also has a synchronous orbit, thus always showing the same "face".

Answers

1. $A = 1.68 \text{ m/s}^2$, $B = 1.95 \text{ m/s}^2$
2. $r = 1659 \text{ m}$
3. a) 1860 N b) $T = 1.74 \text{ h}$ c) $h = 1.7 \times 10^7 \text{ m}$
4. $T_P/T_Q = 0.544$
5. a) 679 N b) $7.31 \times 10^7 \text{ m}$ c) $5.69 \times 10^{26} \text{ kg}$