Scalar Product, Work and Kinetic Energy

1. What is the scalar product between the following vectors:
   a) $\vec{A} = 3\hat{i} - 2\hat{j}$ and, $\vec{B} = 4\hat{i} - 4\hat{j}$
   b) $\vec{A} = -2\hat{i} + 4\hat{j}$ and, $\vec{B} = 3\hat{i} - 4\hat{j}$
   c) $\vec{A} = \hat{i} - 2\hat{j} + 2\hat{k}$ and, $\vec{B} = 3\hat{j} + 4\hat{k}$

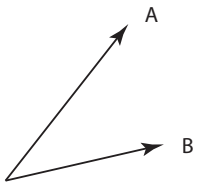
2. Using the definition of the scalar product, find the angles between the same vectors as in the previous question:
   a) $\vec{A} = 3\hat{i} - 2\hat{j}$ and, $\vec{B} = 4\hat{i} - 4\hat{j}$
   b) $\vec{A} = -2\hat{i} + 4\hat{j}$ and, $\vec{B} = 3\hat{i} - 4\hat{j}$
   c) $\vec{A} = \hat{i} - 2\hat{j} + 2\hat{k}$ and, $\vec{B} = 3\hat{j} + 4\hat{k}$

3. Use the scalar product to prove the law of cosines for a triangle: $c^2 = a^2 + b^2 - 2ab\cos\theta$, where a, b, and c are the lengths of the sides of a triangle and θ is the angle opposite side c.

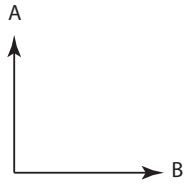
4. Using the three arbitrary vectors $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$, $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$, and $\vec{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$, show that the scalar product of two vectors is distributive: $\vec{A} \cdot (\vec{B} + \vec{C}) = (\vec{A} \cdot \vec{B}) + (\vec{A} \cdot \vec{C})$.

5. A 0.5 kg mass on a 1 m long string swings in a circle on a horizontal, frictionless table at a steady speed of 2 m/s.
   a) How much work does the tension in the string do on the mass during one revolution? ***Explain.***
   b) Is your answer for part a) consistent with the work-kinetic energy theorem? ***Explain.***

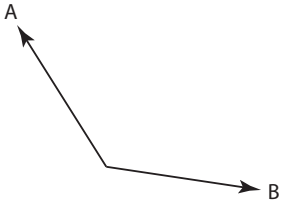
6. For each pair of vectors, is the sign of the scalar product, positive (+), negative (-), or zero (0)?



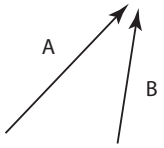
(a)



(b)



(c)



(d)

7. An automobile of mass 10^3 kg moves at 1.0 km/h = 0.28 m/s.
   a) What is its kinetic energy?
   b) At what speeds must a person of mass 80 kg and a bullet of mass 10 g move to have the same kinetic energy as the automobile?
   c) What would the speed of the automobile be if the kinetic energy doubled?

8. A force $\vec{F} = -3.1\hat{i} + 2.7\hat{j}$ N is used to displace an object of mass 17 kg by an amount $\vec{r} = 0.50\hat{i} - 0.75\hat{j}$ m. What is the work done by the force on this object?

Answers

1. a) 20, b) -22, c) 2

2. a) 11.3˚, b) 169.7˚, c) 82.3˚

3. ***Hint:*** $\vec{a}$, $\vec{b}$, and $\vec{c}$ form three sides of an arbitrary triangle, and can be seen as three vectors having the following relationship: $\vec{a} - \vec{b} = \vec{c}$. Then use the fact that $\vec{c} \cdot \vec{c} = |\vec{c}|^2$ ...

4. Calculate each side explicitely. They must be equal.

5. I'm not telling you...

6. a) +, b) 0, c) –, d) +

7.
   a) 39.2 J

   b) 3.56 km/h, 319 km/h

   c) 1.42 km/h

8. W = -3.575 J