Lab #7
Moment of Inertia

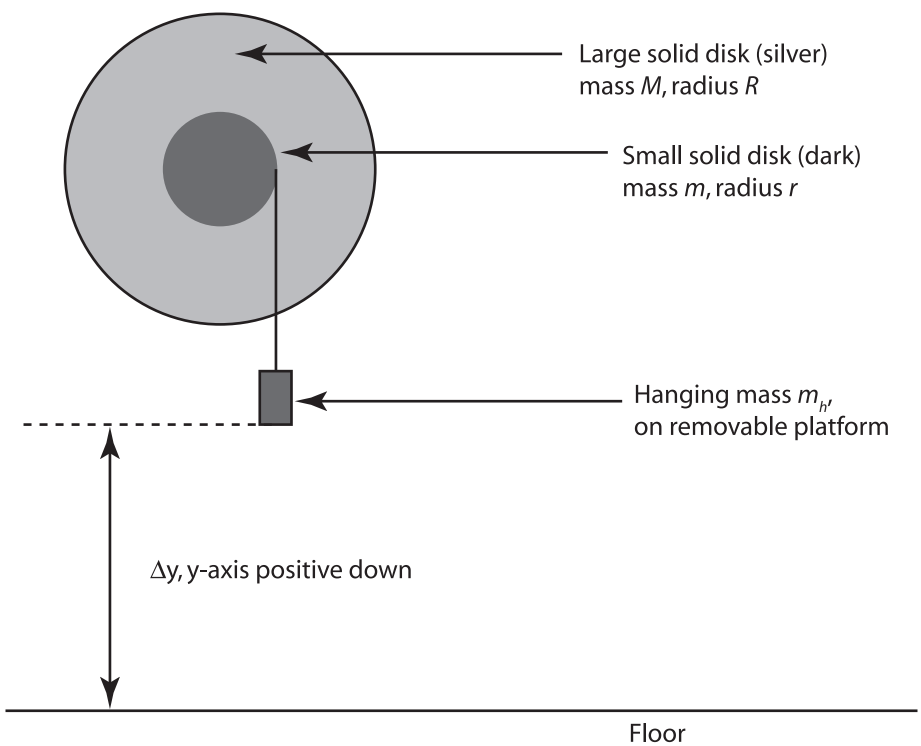
1. Objectives

To determine the moment of inertia of an arrangement of two solid discs as well as the frictional torque on their axle (axis of rotation).

2. Equipment

- Inertia discs
- masses
- stopwatch
- strings
- meter sticks
- vernier calipers
- rulers

3. Set-up



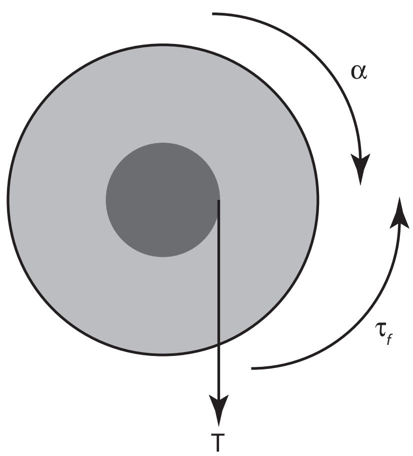
4. Theory

The total moment of inertia of the complete arrangement of disks (the pulley) is the sum of the individual moments of inertia:

$$I = \frac{MR^2}{2} + \frac{mr^2}{2}.$$

From the free body diagram of the hanging mass it is possible to show that the tension in the string must be given by $T = m_h(g-a)$, where a is the magnitude of the vertical acceleration. This acceleration can be obtained from the kinematics by measuring the time it takes m_h to fall (starting at rest) a predetermined vertical displacement Δy. With the y-axis taken to be positive downwards, $\Delta y = \frac{1}{2}at^2$.

For a realistic pulley, some frictional torque will always be present on the axle. Frictional torques that tend to reduce the acceleration, oppose the direction of spin of an object.



$$\sum \tau = I\alpha, \text{ let } \alpha > 0 \text{ clockwise}$$

$$rT - \tau_f = I\alpha$$

$$rT = \tau_T = I\alpha + \tau_f$$

$$\alpha = a/r$$

As can be seen from the above figure, a graph of τ_T versus the magnitude of α will yield a straight line of slope I and intercept τ_f if the frictional torque is constant. The assumption of a constant frictional torque is valid as long as the tension in the string is much less than the total weight of the pulley.

5. Procedure

1) Select one of the pulley setups and take note of the mass written on the pulley. This mass is the mass of the total pulley system (large silver disk M, plus small dark disk m). Assume that the mass of the small disk is 70.0 g.
2) Measure the radius of the large disk R, and the radius of the small disk r, evaluate the uncertainties on these measurements.
3) Tie a loop over the pin in the small disk. Wind a length of string in the proper direction, long enough to bring an attached mass to the floor. Measure the displacement Δy from the horizontal platform to the ground (y-axis positive down).
4) Attach the hanger, m_h, to the other end of the string. Secure the hanger to the platform. Adjust the pulley so the string is taut.
5) When ready, release the platform and simultaneously start the stopwatch. Stop the stopwatch at the instant the mass hits the floor and take note of the time. Ensure that, throughout the fall, the mass does not touch anything. Try to minimize the twisting of the string.

6) Repeat the experiment adding masses on the hanger, with masses ranging from 10g to about 150g.

6. Analysis (in lab report)

1) Use the equation $\Delta y = \frac{1}{2}at^2$ to find the value of the acceleration *a* corresponding to each mass. From this value, obtain the magnitude of the angular acceleration $\alpha = a/r$.
2) Make a table of all measured and calculated data including uncertainties: hanging mass, time, acceleration, angular acceleration, tension, and torque.

3) Plot a graph of τ versus α. The graph should be a straight line. Use LINEST function of Excel to determine the slope and intercept, and the uncertainties on these values.
4) What do these values represent?
5) The following uncertainties must be evaluated, justified or calculated in your report: the uncertainty on measured time, the distance fallen and the radii of the pulleys. The uncertainties on the expected value of moment of inertia, and on the experimental moment of inertia, calculated from the slope of the graph.
6) Compare the value of I obtained from the graph to the theoretical value $I = \frac{MR^2}{2} + \frac{mr^2}{2}$. Does the theoretical value fall within the range of experimental uncertainties?